ANSWERS & SOLUTIONS FOR AIEEE - 2010

CODE - A

PHYSICS , CHEMISTRY & MATHEMATICS

PHYSICS

Directions: Questions number 1-3 are based on the following paragraph
An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I,$ where μ_0 *and* μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

1. The initial shape of the wavefront of the beam is
   1) planar 2) convex
   3) concave 4) convex near the axis and concave near the periphery
   Ans: (3)
   Sol : Conceptual
2. The speed of light in the medium is
   1) maximum on the axis of the beam 2) minimum on the axis of th beam
   3) the same everywhere in the beam 4) directly proportional to the intensity I
   Ans : (2)
   Sol :Conceptual
3. As the beam enters the medium , it will
   1) travel as a cylindrical beam 2) diverge
   3) converge 4) diverge near the axis and converge near the periphery
   Ans : (3)
   Sol : Conceptual

Directions: Questions number 4-5 are based on the following paragraph.
A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c.

4. The speed of daughter nuclei is

   1) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$ 2) $c\frac{\Delta m}{M+\Delta m}$ 3) $c\sqrt{\frac{2\Delta m}{M}}$ 4) $c\sqrt{\frac{\Delta m}{M}}$

   Ans : (3)

   Sol: $\Delta mc^2 = \frac{1}{2}Mv^2$

   $V = \sqrt{\frac{2\Delta mc^2}{M}}$

   $V = \sqrt{\frac{2\Delta m}{M}}c$

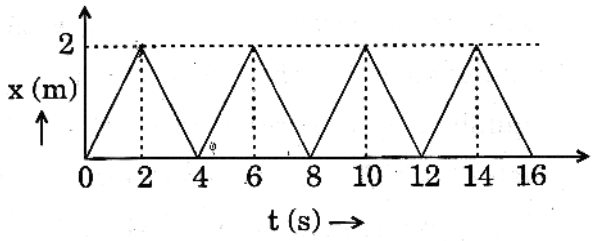
5. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then
   1) $E_1 = 2E_2$ 2) $E_2 = 2E_1$ 3) $E_1 > E_2$ 4) $E_2 > E_1$
   Ans : (4)
   Sol : Conceptual

Directions : *Questions number 6-7 contain Statement-1 and Statement -2. Of the four choices given after the statements, choose the one that best describes the two statements.*

6. **Statement -1** : When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays , both V_0 and K_{max} increase
Statement-2 : Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light
1) Statement-1 is true, Statement-2 is false
2) Statement -1 is true, Statement -2 is true, Statement-2 is the correct explanation of State ment-1
3) Statement -1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement -1.
4) Statement -1 is false,Statement -2 is true.
Ans : (1)
Sol : Conceptual

7. **Statement -1** : Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.
Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.
1) Statement-1 is true, Statement-2 is false
2) Statement -1 is true, Statement -2 is true, Statement-2 is the correct explanation of State ment-1
3) Statement -1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement -1.
4) Statement -1 is false,Statement -2 is true.
Ans : (2)
Sol :Conceptual

8. The figure shows the position -time (x -t) graph of one-dimensional motion of a body of mass 0.4kg. The magnitude of each impulse is



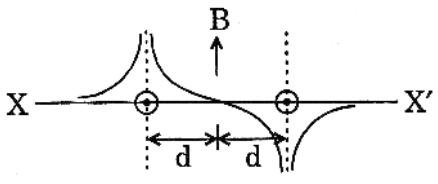
1) 0.2Ns 2) 0.4 Ns 3) 0.8 Ns 4) 1.6 Ns
Ans : (3)

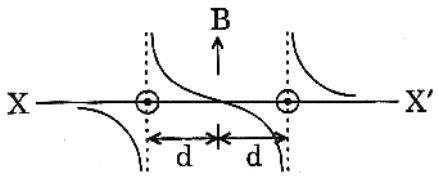
Sol : $V = \frac{s}{t} = \frac{2}{2} = 1$

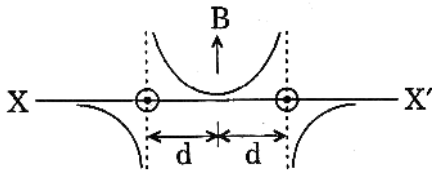
$I = 2mv$

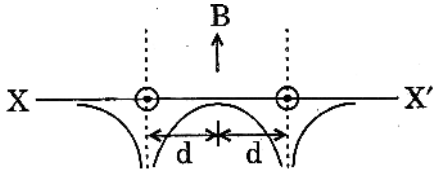
$= 0.4 \times 2 = 0.8 N \sec$

9. Two long parallel wires are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX′ is given by

1) 

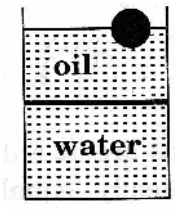
2) 

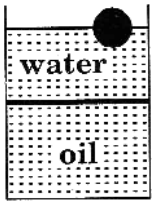
3) 

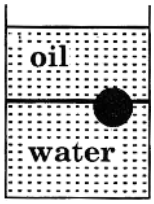
4) 

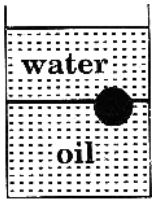
Ans : (2)
Sol :Conceptual

10. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ *with* ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible . If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position ?

1) 

2) 

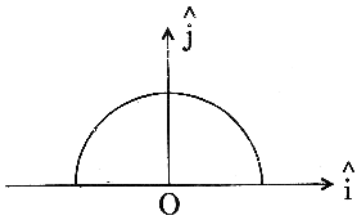
3) 

4) 

Ans : (3)
Sol : Conceptual

11. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it.

The net field $\vec{E}$ at the centre O is



1) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ 2) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ 3) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ 4) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

Ans : (4)

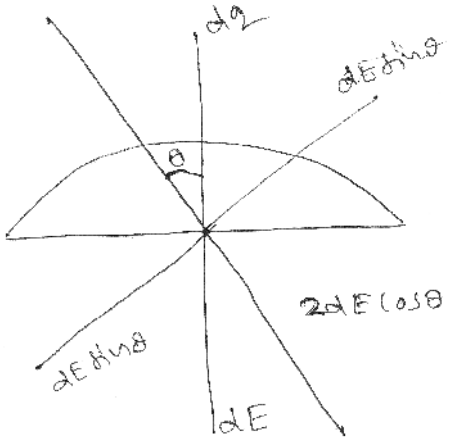
Sol :

$$E = \int_0^{\pi/2} 2dE\cos\theta$$

$$E = \int_0^{\pi/2} \frac{2dq\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$E = \int_0^{\pi/2} 2\times\frac{q\ rd\theta\cos\theta}{\pi r\ 4\pi 2r^2}$$

$$E = \frac{-q}{2\pi^2\varepsilon_0 r^2}J$$



12. A diatomic ideal gas is used in a Carnot engine as the working substance . If during the adiabatic expansion part of the cycle the volume of the gas increases from V to 32V, the efficiency of the engine is

1) 0.25 2) 0.5 3) 0.75 4) 0.99

Ans :(3)

Sol :

$\gamma = 7/5$ $\quad T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$\frac{T_1}{T_2} = \left[\frac{32V}{V}\right]^{7/5-1}$$

$$= \left[2^5\right]^{2/5}$$

$$\frac{T_1}{T_2} = 4$$

$$\eta = 1 - T_2/T_1 = 1 - 1/4 = \frac{3}{4} = 0.75$$

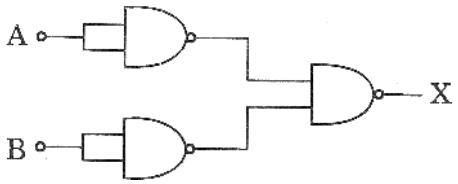
13. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are

1) 4,4,2 2) 5,1,2 3) 5,1,5 4) 5,5,2

Ans : (2)

Sol : Conceptual

14. The combination of gates shown below yields



1) NAND gate 2) OR gate 3) NOT gate 4) XOR gate

Ans : (2)

Sol : $\overline{\overline{A}.\overline{B}} = A + B \quad \rightarrow OR\ gate$

15. If a source of power 4kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called

1) $\gamma - rays$ 2) X-rays 3) ultraviolet rays 4) microwaves

Ans :(2)

Sol : $P = \frac{nhc}{\lambda} \Rightarrow \lambda = \frac{nhc}{p}$

$$= \frac{10^{20}\times6.6\times10^{-34}\times3\times10^{8}}{4\times10^{3}} = 50A^0$$

X Rays

16. A radioactive nucleus (initial mass number A and atomic number Z) emits $3\ \alpha$ – particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

1) $\frac{A-Z-4}{Z-2}$ 2) $\frac{A-Z-8}{Z-4}$ 3) $\frac{A-Z-4}{Z-8}$ 4) $\frac{A-Z-12}{Z-4}$

Ans :(3)

Sol : $Z^{X^A} \rightarrow Z-6^{X^{A-12}} + 2e^0_{+1}$

Atomic number = Z-4

Mass number = A-12

$$\frac{no.of\ \ neutrons}{no.of\ \ protons} = \frac{A-12-(Z-8)}{Z-8} = \frac{A-Z-4}{Z-8}$$

17. Let there be a spherically symmetric charge distribution with charge density varying as

$\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto r=R, and $\rho(r) = 0$ for r > R, where r is the distance from the origin.

The electric field at a distance r(r < R) from the origin is given by

1) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ 2) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ 3) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ 4) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

Ans : (3)

Sol :

$$\frac{dQ}{dV} = \rho_0\left[\frac{5}{4} - \frac{r}{R}\right]$$

$$dQ = \rho_0\left[\frac{5}{4} - \frac{r}{R}\right]4\pi r^2 dr$$

$$= 4\pi\rho_0\left[\frac{5r^3}{12} - \frac{r^4}{4R}\right]$$

$$Q = \frac{4\pi\rho_0 r^3}{4}\left[\frac{5}{3} - \frac{r}{R}\right]$$

$$E.dS = \frac{Q}{\varepsilon_0}$$

$$E = \frac{\pi\rho_0 r^3}{4\pi r^2}\left[\frac{5}{3} - \frac{r}{R}\right]$$

$$E = \frac{\rho_0 r}{4\varepsilon_0}\left[\frac{5}{3} - \frac{r}{R}\right]$$

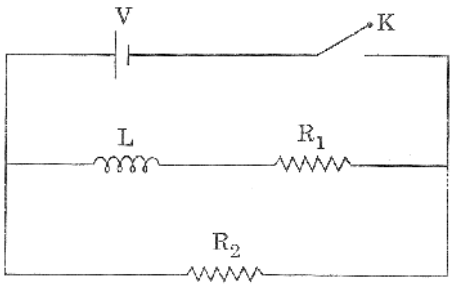
18. In a series LCR circuit $R = 200\Omega$ and the voltage and the frequency of the main supply is 220V and 50Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30^0. On taking out the inductor from the circuit the current leads the voltage by 30^0. The power dissipated in the LCR circuit is

1) 242W 2) 305W 3) 210W 4) Zero W

Ans : (1)

Sol : $P = \frac{V^2}{R} = \frac{220\times 220}{200} = 242W$

19. In the circuit shown below, the key K is closed at t=0. The current through the battery is



1) $\frac{V(R_1+R_2)}{R_1R_2} at\ t=0\ and\ \frac{V}{R_2} at\ t=\infty$ 2) $\frac{VR_1R_2}{\sqrt{R_1^2R_2^2}} at\ t=0\ and\ \frac{V}{R_2} at\ t=\infty$

3) $\frac{V}{R_2} at\ t=0\ and\ \frac{V(R_1+R_2)}{R_1R_2} at\ t=\infty$ 4) $\frac{V}{R_2} at\ t=0\ and\ \frac{VR_1R_2}{\sqrt{R_1^2R_2^2}} at\ t=\infty$

Ans : (3)
Sol : Conceptual

20. A particle is moving with velocity $\vec{\upsilon}=K\left(y\hat{i}+x\hat{j}\right)$, where K is a constant . The general equation for its path is

1) $y^2=x^2+\text{constant}$ 2) $y=x^2+\text{constant}$ 3) $y^2=x+\text{constant}$ 4) xy = constant

Ans : (1)

Sol :

$$\overline{V}=K\left(y\hat{i}+x\hat{j}\right)$$

$$\frac{d\bar{r}}{dt}=Ky\hat{i}+Kx\hat{j}$$

$$\Rightarrow \bar{r}=Kyt\hat{i}+Kxt\hat{j}+c$$

$$\Rightarrow r^2=K^2y^2t^2+K^2x^2t^2+\text{constant}$$

$$\left(x^2+y^2\right)=K^2y^2t^2+K^2x^2t^2+\text{constant}$$

$$y^2\left[1-K^2t^2\right]=x^2\left[K^2t^2-1\right]+\text{constant}$$

$$y^2=\frac{x^2\left[K^2t^2-1\right]}{\left[1-K^2t^2\right]}+\text{constant}$$

$$=-x^2+current=x^2+\text{constant}$$

$$\therefore y^2=x^2+\text{constant}$$

21. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be

1) 2 2) 1 3) $\frac{1}{2}$ 3) $\frac{1}{4}$

Ans : (4)

Sol : $\frac{U_0}{2} = U_0 \times \left(e^{-t_1/Rc}\right)^2$

$\frac{Q}{4} = Q_0\left[e^{-t_2/Rc}\right]$

$\frac{t_1}{t_2} = \frac{1}{4}$

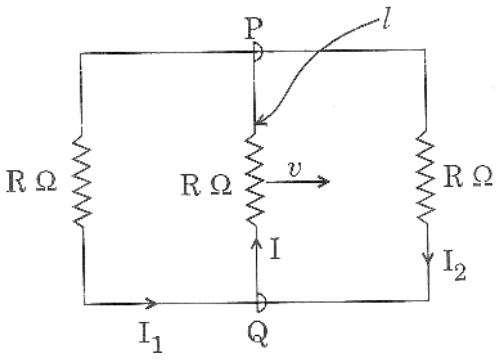
22. A rectangular loop has a sliding connector PQ of length *l* and resistance R Ω and it is moving with a speed *v* as shown . The set-up is placed in a uniform magnetic field going into the plane of the paper . The three currents I_1,I_2 and I are

1) $I_1 = I_2 = \frac{Bl\upsilon}{6R}, I = \frac{Bl\upsilon}{3R}$

2) $I_1 = -I_2 = \frac{Bl\upsilon}{R}, I = \frac{2Bl\upsilon}{R}$

3) $I_1 = I_2 = \frac{Bl\upsilon}{3R}, I = \frac{2Bl\upsilon}{3R}$

4) $I_1 = I_2 = I = \frac{Bl\upsilon}{R}$



Ans : (3)

Sol : Total Resistance $(R') = \frac{R}{2} + R = \frac{3}{2}R$

$I = \frac{Blv}{R'} = \frac{2Blv}{3R}$

$I_1 = I_2 = \frac{Blv}{3R}$

23. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by y = 0.02(m) $\sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{X}{0.50(m)}\right)\right]$. The tension in the string is

1) 6.25 N 2) 4.0 N 3) 12.5 N 4) 0.5 N

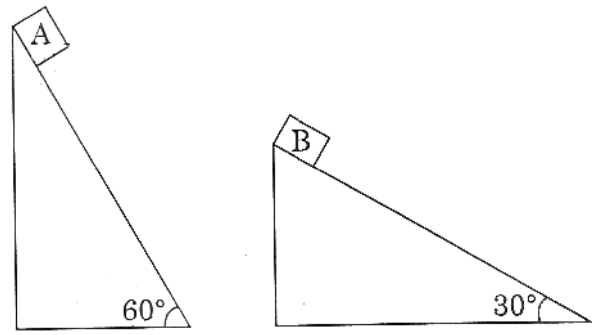
Ans : (1)

Sol : $V = \frac{\omega}{K} = \sqrt{\frac{T}{m}}$

$V = \frac{0.5}{0.04} = \sqrt{\frac{T}{0.04}}$

$T = 6.25N$

24. Two fixed frictionless inclined planes making an angle 30^0 and 60^0 with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. what is the relative vertical acceleration of A with respect to B?



1) 4.9 ms^{-2} in vertical direction
2) 4.9 ms^{-2} in horizontal direction
3) 9.8 ms^{-2} in vertical direction
4) Zero

Ans : (1)

Sol : $g_1 = g\sin 60^0 \cos 30^0$

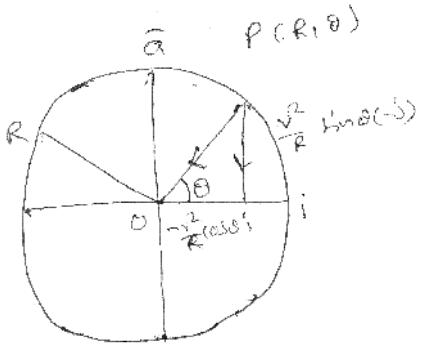
$g_2 = g\sin 30^0 \cos 60^0$

$g = g_1 - g_2 = \frac{g}{2} = 4.9 m/\sec^2$ *vertical direction*

25. For a particle in uniform circle motion, the acceleration $\vec{a}$ at a point $P(R,\theta)$ on the circle of radius R is (Here θ is measured from the X-axis)

1) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$
2) $-\frac{v^2}{R}\cos\theta\hat{i} + \frac{v^2}{R}\sin\theta\hat{j}$
3) $-\frac{v^2}{R}\sin\theta\hat{i} + \frac{v^2}{R}\cos\theta\hat{j}$
4) $-\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$

Ans : (4)



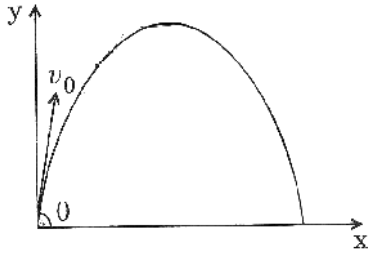
Sol : $a_c = \frac{V^2}{R}$

from diagram

$a_c = \frac{-V^2}{R}\cos\theta\hat{i} - \frac{-V^2}{R}\sin\theta\hat{j}$

26. A small particle of mass m is projected at an angle θ with the X-axis with an intial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0\sin\theta}{g}$,the angular momentum of the particle is

1) $\frac{1}{2}mg\,v_0 t^2\cos\theta\hat{i}$
2) $-mg\,v_0 t^2\cos\theta\,\hat{j}$
3) $mg\,v_0 t\cos\theta\hat{k}$
4) $-\frac{1}{2}mg\,v_0 t^2\cos\theta\hat{k}$



Where $\hat{i}, \hat{j}$ *and* $\hat{k}$ are unit vectors along x, y and z-axis respectively.

Ans : (4)

Sol :

$$t < \frac{V_0 \sin\theta}{g}$$

$$V = V_0 \cos\theta \hat{i} + (V_0 \sin\theta - gt)\,\hat{j}$$

$$r = V_0 \cos\theta\, t\,\hat{i} + \left(V_0 \sin\theta\, t - \frac{1}{2}gt^2\right)\hat{j}$$

$$L = \vec{r} \times mv = m \begin{bmatrix} i & j & k \\ V_0 \cos\theta t & V_0 \sin\theta\, t - \frac{1}{2}gt^2 & 0 \\ V_0 \cos\theta & V_0 \sin\theta - gt & 0 \end{bmatrix}$$

$$= mgt^2 V_0 \cos\theta \left(-1 + \frac{1}{2}\right)\hat{k}$$

$$= -\frac{1}{2} mgV_0 t^2 \cos\theta \hat{k}$$

27. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30^0 with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remans the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is

1) 1 2) 4 3) 3 4) 2

Ans : (4)

Sol : $K = \dfrac{d}{d-p}$ $\quad d = 1.6$

$p = 0.8$

$$= \frac{1.6}{1.6 - 0.8} = \frac{1.6}{0.8}$$

$k = 2$

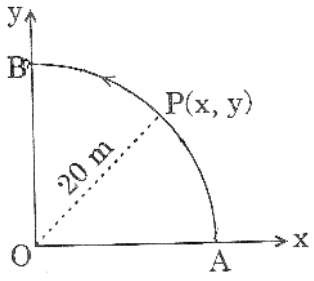
28. A Point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length s = t^3 + 5, where S is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when t = 2 S is nearly

1) 14 m/s^2

2) 13 m/s^2

3) 12 m/s^2

4) 7.2 m/s^2



Ans : (1)

Sol : $V = \frac{ds}{dt} = 3t^2$

$a_t = \frac{d^2s}{dt^2} = 6t = 12$

$a_n = \frac{V^2}{r} = \frac{(3\times 4)^2}{20} = \frac{144}{20}$

$a = \sqrt{a_n^2 + a_t^2} = 14m/\sec^2$

29. The pontential energy function for the force between two atoms in a diatomic molecule is approximately given by U (X) = $U(X) = \frac{a}{X^{12}} - \frac{b}{X^6}$, where a and b are constants and X is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(X = \infty) - U_{at\ equilibrium}], D\ is$

1) $\frac{b^2}{6a}$ 2) $\frac{b^2}{2a}$ 3) $\frac{b^2}{12a}$ 4) $\frac{b^2}{4a}$

Ans : (4)

Sol:

$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$

$D = \left[V(x = \infty) - V_{at}\ equilibrium\right]$

$dU = 0$

$= \frac{-12a}{x^{13}} + \frac{6b}{x^7} = 0$

$\frac{2a}{x^6} = b$

$x^6 = \frac{2a}{b}$

$D = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{(2a/b)} = \frac{ab^2}{4a^2} - \frac{b^2}{2a}$

$D = \frac{-b^2}{4a}$

$D = 0 - \left(-\frac{b^2}{4a}\right) = \frac{b^2}{4a}$

30. Two conductors have the same resistance at 0^0C but their temperature coefficients of resistance are $\alpha_1 + \alpha_2$. The respective temperature coefficients of their series and parallel combinations are nearly

1) $\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + \alpha_2}{2}$ 2) $\frac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$ 3) $\alpha_1 + \alpha_2, \frac{\alpha_1 + \alpha_2}{2}$ 4) $\alpha_1 + \alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2}$

Ans : (1)

Sol : In series

$R_0 = R_1 + R_2$

$R_t = R_1^{'} + R_2^{'}$

$= R_1 + R_{1\alpha_1 t} + R_2 + R_{2\alpha_1 t}$

$= (R_1 + R_2) + t[R_1\alpha_1 + R_2\alpha_2].....(i)$

But $R_1 = R_0 + R_0\alpha t$

$= (R_1 + R_2) + (R_1 + R_2)\alpha t ----(ii)$

From $(i)\ \&\ (ii)$

$\alpha = \dfrac{(R_1\alpha_1 + R_2\alpha_2)}{(R_1 + R_2)}$

$= \dfrac{\alpha_1 + \alpha_2}{2}(\Theta R_1 = R_2)$

In parallel

$R_0 = \dfrac{R}{2}$

$R_t = \dfrac{R[1+\alpha_1 t]R[1+\alpha_2 t]}{R\{[1+\alpha_1 t]+[1+\alpha_2 t]\}}$

$= \dfrac{R(1+\alpha_1 t)(1+\alpha_2 t)}{[2+\alpha_1 t+\alpha_2 t]} \qquad ---(ii)$

But $R_t = \dfrac{R}{2}[1+\alpha t] ----(ii)$

From $(i)\ \&\ (ii)(1+\alpha t)$

$= \dfrac{2[1+\alpha_1 t](1+\alpha_2 t)}{[2+\alpha_1 t+\alpha_2 t]}$

$\alpha t = \dfrac{2[1+\alpha_1 t+\alpha_2 t+\alpha_1\alpha_2 t^2]}{[2+\alpha_1 t+\alpha_2 t]} - 1$

$= \dfrac{\alpha_1 t+\alpha_2 t+\alpha_1\alpha_2 t^2}{(2+\alpha_1 t+\alpha_2 t)}$

$= \dfrac{t(\alpha_1+\alpha_2+\alpha_1\alpha_2 t)}{2+(\alpha_1+\alpha_2)t}$

$\Rightarrow \alpha = \dfrac{\alpha_1+\alpha_2+\alpha_1\alpha_2 t}{2+(\alpha_1+\alpha_2)t}; At\ t = 0,$

$\alpha = \dfrac{\alpha_1+\alpha_2}{2}$

CHEMISTRY

31. In aqueous solution the ionization constants for carbonic acid are $K_1 = 4.2\times10^{-7}$ and $K_2 = 4.8\times10^{-11}$. Select the correct statement for a saturated 0.034 M solution of the carbonic acid.

1) The concentration of H^+ is double that of CO_3^{2-}.

2) the concentration of CO_3^{2-} is 0.034 M.

3) The concentration of CO_3^{2-} is greater than that of HCO_3^-

4) The concentrations of H^+ and HCO_3^- are approximately equal.

Ans : 4

32. Solubility product of silver bromide is 5.0×10^{-13}. The quantity of potassium bromide (molar mass taken as 120 gm mol^{-1}) to be added to 1 litre of 0.05 M solution of silver nitrate to start the precipitation of AgBr is

1) $5.0\times10^{-8}\,g$ 2) $1.2\times10^{-10}\,g$ 3) $1.2\times10^{-9}\,g$ 4) $6.2\times10^{-5}\,g$

Ans : 3

Sol :

$$k_{sp} = S^1\times C$$

$$\frac{5\times10^{-13}}{5\times10^{-2}} = C$$

$$C = 10^{-11}\,mole/lt$$

$$\therefore C = \frac{lit}{120}\times\frac{1000}{1000}$$

$$10^{-11}\times120 = lit$$

$$lit = 1.2\times10^{-9}\,gms$$

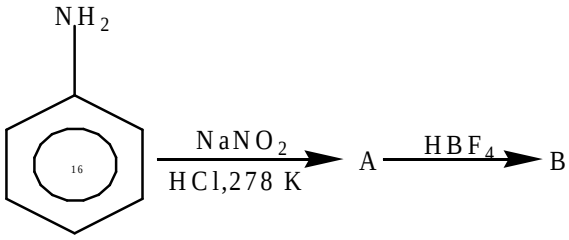
33. The correct sequence which shows decreasing order of the ionic radii of the elements is

1) $O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$ 2) $Al^{3+} > Mg^{2+} > Na^+ > F^- > O^{2-}$

3) $Na^+ > Mg^{2+} > Al^{3+} > O^{2-} > F^-$ 4) $Na^+ > F^- > Mg^{2+} > O^{2-} > Al^{3+}$

Ans:1

Sol: Size $\alpha - \frac{-ve\ ch\arg e}{+ve\ ch\arg e}$ **(for iso electronic ions)** $\therefore O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$

34. In the chemical reactions, 

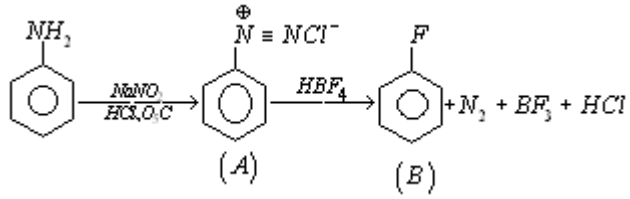
the compounds 'A' and 'B' respectively are

1) nitrobenzene and chlorobenzene 2) nitrobenzene and fluorobenzene

3) phenol and benzene

4) benzene diazonium chloride and fluorobenzene

Ans:4

Sol: 

35. If 10^{-4} dm^3 of water is introduced into a 1.0 dm^3 flask at 300 K, how many moles of water are in the vapour phase when equilibrium is estabilished?

(Given: Vapour pressure of H_2O at 300 K is 3170 Pa; R = 8.314 J K^{-1} mol^{-1})

1) 1.27×10^{-3} *mol* 2) 5.56×10^{-3} *mol* 3) 1.53×10^{-2} *mol* 4) 4.46×10^{-2} *mol*

Ans : 1

Sol : $1dm^3 = 10^{-3}m^3$

$PV = nRT$

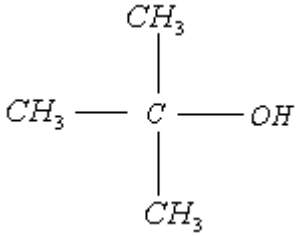
$\frac{10^{-3}\times3170}{8.314\times300} = n$

36. From amongst the following alcohlols the one that would react fatest with conc. HCl and anhydrous $ZnCl_2$, is

1) 1-Butanol 2) 2-Butanol

3) 2-Methylpropan-2-o1 4) 2-Methylpropanol

Ans:3

Sol: Tertiary alcohol reacts fastest with conc. HCl and Zn Cl_2 

Order of reactivityof alcohols $3^0 > 2^0 > 1^0$

(It is a tertiary alcohol)

37. If sodium sulphate is considered to be completely dissociated into cations and anions in aqueous solution, the change in freezing point of water $\left(\Delta T_f\right)$, when 0.01 mol of sodium sulphate is dissolved in 1 kg of water, is $\left(K_f = 1.86K\ kg\ mol^{-1}\right)$

1) 0.0186 K 2) 0.0372 K 3) 0.0558 K 4) 0.0744 K

Ans : 3

Sol : $Na_2SO_4 \rightarrow 2Na^+ + SO_4^{2-}$

$\Delta H_f = i\ k_f.m.$

$= 3\times1.86\times0.01$

$= 0.558$

38. Three reactions involving $H_2PO_4^-$ are given below:

i) $H_3PO_4 + H_2O \rightarrow H_3O^+ + H_2PO_4^-$

ii) $H_2PO_4^- + H_2O \rightarrow HPO_4^{2-} + H_3O^+$

iii) $H_2PO_4^- + OH^- \rightarrow H_3PO_4 + O^{2-}$

In which of the above does $H_2PO_4^-$ act as an acid?

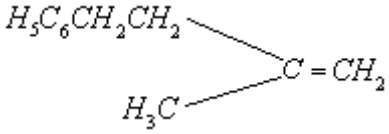
1) (i) only 2) (ii) only 3) (i) and (ii) 4) (iii) only

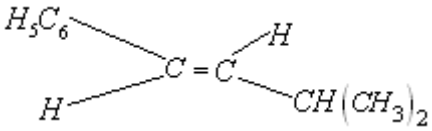
Ans : 2

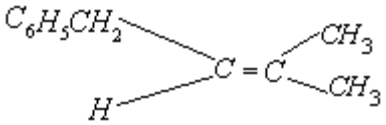
Sol : $H_2PO_4^- + H_2O \rightarrow HPO_4^{2-} + H_3O^+$

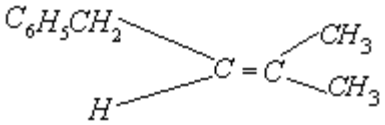
$H_2PO_4^- \rightleftharpoons H^+ + HPO_4^{2-}$

39. The main product of the following reaction is $C_6H_5CH_2CH(OH)CH(CH_3)_2 \xrightarrow{conc.H_2SO_4} ?$

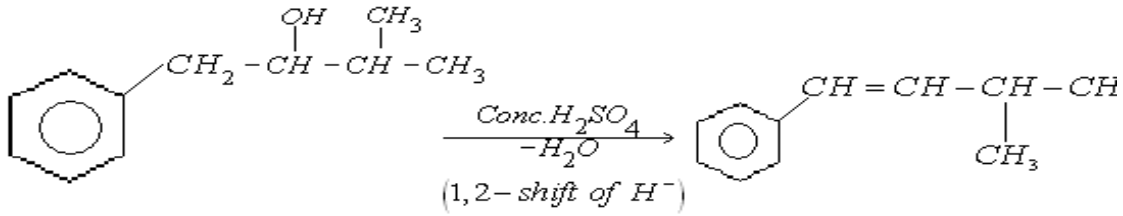
1) 

2) 

3) 

4) 

Ans : 2

Sol : 

(Transform) stability is due to conjugation and trans form is more stable than cis form.

40. The energy required to break one mole of Cl - Cl bonds Cl_2 is 242 kJ mol^{-1}. The longest wavelength of light capable of breaking a single Cl - Cl bond is

$(c = 3\times10^8 ms^{-1} and\ N_A = 6.02\times10^{23} mol^{-1})$

1) 494 cm 2) 594 cm 3) 640 nm 4) 700 nm

Ans : 1

Sol : The nergy required to break 6.02×10^{23} Of Cl - Cl $242\times10^3 J/mol$

The nergy required to break one Cl - Cl bond?

$$= \frac{242\times10^3}{6.02\times10^{23}} = 40.19\times10^{-20} J$$

$$\therefore E = \frac{LC}{\lambda} \Rightarrow \lambda\frac{hc}{E} = \frac{6.625\times10^{-34}\times3\times10^8}{40.19\times10^{-20}}$$

$$= 0.494\times10^{-6} = 494\times10^{-9} m = 494nm$$

41. 29.5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method and the evolved ammonia was absorbed in 20 mL of 0.1 m HCl solution. The excess of the acid required 15 mL of 0.1 M NaOH solution of complete neutralization. The percentage of nitrogen in the compound is

1) 29.5 2) 59.0 3) 47.4 4) 23.7

Ans : 4

Sol : kjeldahl's method % N = $\frac{1.4\times N(acid)V(Acid\ used)}{W(organic\ compound)}$

Excess of acid

$NaVa = NbVb \quad 0.1 \times Va = 0.1 \times 15 \quad Va = 15ml$

$\therefore V\,acid\,used = 5ml\ (20-15)$

$\%N = \dfrac{1.4 \times 0.1 \times 5}{29.5 \times 10^{-3} g} = 23.7\%$

42. Ionisation energy of He^{+} is 19.6×10^{-18} $J\ atom^{-1}$. The energy of the first stationary state (n = 1) of Li^{2+} is

1) 8.82×10^{-17} $J\ atom^{-1}$ 2) 4.41×10^{-16} $J\ atom^{-1}$

3) -4.41×10^{-17} $J\ atom^{-1}$ 4) -2.2×10^{-15} $J\ atom^{-1}$

Ans : 3

Sol : $\dfrac{(I.E)_{He^{+}}}{(I.E)_{Li^{+2}}} = \dfrac{13.6 \times 4}{13.6 \times 9} \Rightarrow (I.E)_{Li^{+2}} = \dfrac{9}{4} \times (I.E)_{He^{+}}$

$= \dfrac{9}{4} \times 19.6 \times 10^{-18} = 44.1 \times 10^{-18} J$

$= -4.41 \times 10^{-17}\ J$

$\therefore$ Energy of first orbit in $Li^{+2} = -4.41 \times 10^{-17}$ J

43. On mixing heptane and octane form and ideal solution. At 373 K, the vapour pressures of the two liquid components (heptane and octane) are 105 kPa and 45 kPa respectively. Vapour pressure of the solution obtained by mixing 25.0 g of heptane and 35 g of octane will be (molar mass of heptane = 100 g mol^{-1} and of octane = 114 g mol^{-1})

1) 144.5 kPa 2) 72.0 kPa 3) 36.1 kPa 4) 96.2 kPa

Ans : 2

Sol : $n_{lep\tan e} = \dfrac{25}{100} = 0.25,\ n_{Oc\tan e} = \dfrac{35}{114} = 0.307$

$Total\ mole = 0.25 + 0.307 = 0.557$

$P = 105 \times \dfrac{0.25}{0.557} + 45 \times \dfrac{0.307}{0.557}$

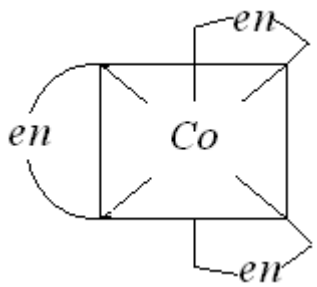
$= 47.12 + 24.8 = 71.92 = 72$

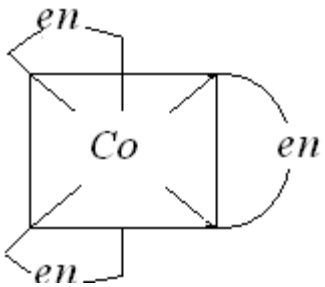
44. Which of the following has an optical isomer ?

1) $\left[Zn(en)_2\right]^{2+}$ 2) $\left[Zn(en)(NH_3)_2\right]^{2+}$ 3) $\left[Co(en)_3\right]^{3+}$ 4) $\left[Co(H_2O)_4(en)\right]^{3+}$

Ans : 3

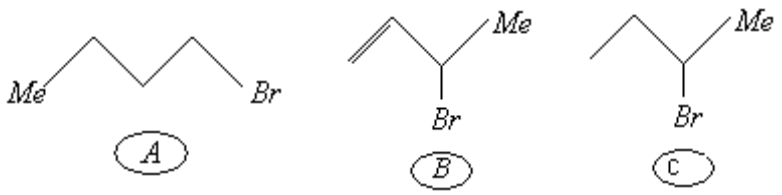
Sol :





Octahedral complex with Formula $\left[M\left(AA\right)_3\right]$ exhibits optical isomerism.

45. Consider the following bromides:

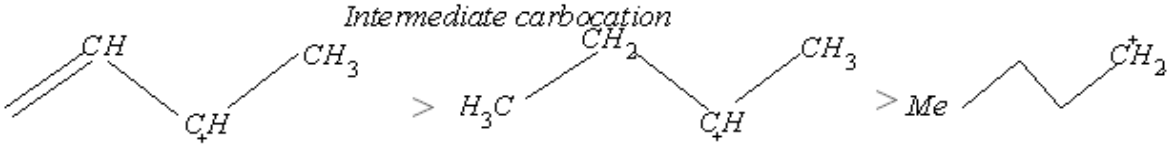


The correct order of S_N1 reactivity is

1) A > B > C 2) B > C > A 3) B > A > C 4) C > B > A

Ans : 2

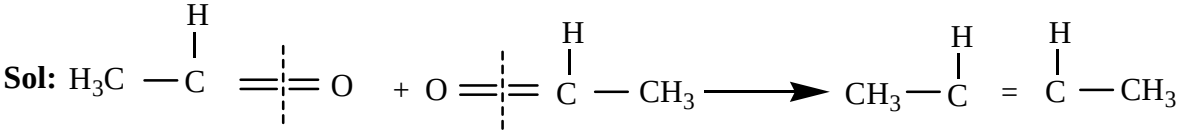
Sol : Rate of S_N1 α stability of



46. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of 44 u. The alkene is

1) ethene 2) propene 3) 1-butene 4) 2-butene

Ans :4

Sol: 

47. Consider the reaction : $Cl_2\left(aq\right)+H_2S\left(aq\right)\rightarrow S\left(s\right)+2H^+\left(aq\right)+2Cl^-\left(aq\right)$

The rate equation for this reaction is rate $=k\left[Cl_2\right]\left[H_2S\right]$

Which of these mechanisms is/are consistent with this rate equation?

A. $Cl_2+H_2S\rightarrow H^++Cl^-+Cl^++HS^-$ (slow)

B. $H_2S\Leftrightarrow +H^++HS^-$ (fast equilibrium) $Cl_2+HS^-\rightarrow 2Cl^-+H^++S$ (slow)

1) A only 2) B only 3) Both A and B 4) Neither A nor B

Ans : 1

Sol : $n=1.27\times10^{-3}mole$ **for A. rate =** $k\left[Cl_2\right]\left[H_2S\right]$

B. rate = $k\left[Cl_2\right]\left[HS^-\right]$ **for** $\left[HS^-\right]$**:-** $k^1=\frac{\left[H^+\right]\left[H.S^-\right]}{H_2S}$ $\left[HS^-\right]=\frac{k^1\left[H_2S\right]}{\left[H^+\right]}=kk^1=k$

$$rate=\frac{k\left[Cl_2\right]\left[H_2S\right]}{H^+}$$

48. The Gibbs energy for the decomposition of Al_2O_3 at 500 ^0C is as follows:

$$\frac{2}{3}Al_2O_3\rightarrow\frac{4}{3}Al+O_2,\Delta_rG=+966\ kJ\ mol^{-1}$$

The potential difference needed for electrolytic reduction of Al_2O_3 at 500 ^0C is at least

1) 5.0 V 2) 4.5 V 3) 3.0 V 4) 2.5 V

Ans : 4

Sol :The charge involved = $4 \times 96,500C$ **. Energy = charge** $\times$ **potential difference**

p.d = $\frac{966 \times 10^3}{4 \times 96,500} = 2.5V$

49. The correct order of increasing basicity of the given conjugate bases $(R = CH_3)$ is

1) $RCO\overline{O} < HC \equiv \overline{C} < \overline{NH_2} < \overline{R}$ 2) $RCO\overline{O} < HC \equiv \overline{C} < \overline{R} < \overline{NH_2}$

3) $\overline{R} < HC \equiv \overline{C} < RCO\overline{O} < \overline{NH_2}$ 4) $RCO\overline{O} < \overline{NH_2} < HC \equiv \overline{C} < \overline{R}$

Ans : 1

Sol: $R-COOH > HC \equiv CH > NH_3 > CH_4 (order\ of\ \ acidic\ \ strength)$

$RCOO^- < HC \equiv \overset{\ominus}{C} < \overset{\ominus}{N}H_2 < \overset{\ominus}{C}H_3$ ($\because$ order of basicity)

50. The edge length of face centered cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is

1) 144 pm 2) 288 pm 3) 398 pm 4) 618 pm

Ans : 1

Sol : $2r_+ + 2r_- = edge\ length$

$2 \times 110 + 2r_- = 508 \ = 2r_- = 508 - 220 = 288$

$r_- \frac{288}{2} = 144\,pm$

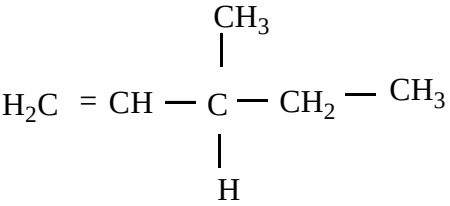
51. Out of the following, the alkene that exhibits optical isomerism is

1) 2-methyl-2pentene 2) 3-methyl-2pentene

3) 4-methyl-1pentene 4) 3-methyl-1pentene

Ans : 4

Sol :



3rd carbon is chiral carbon

52. For a particular reversible reaction at temperature T, ΔH and ΔS were found to be both +ve. If Te is the temperature at equilibrium, the reaction would be spontaneous when

1) $T = T_e$ 2) $T_e > T$ 3) $T > T_e$ 4) T_e is 5 times T

Ans : 3

Sol : $\therefore where\ \Delta H = T\Delta S$

$\Delta G = 0\ at\ T\ \ \Delta G = 0$

$where\ \ T > T_e\ \ then\ \Delta G = -ve$

53. Percentages of free space in cubic close packed structure and in body centered packed structure are respectively

1) 48% and 26% 2) 30% and 26% 3) 26% and 32% 4) 32% and 48%

Ans : 3

Sol : Packing percentage in ccp - 74%

packing percentage in bcc - 68%

$\therefore$ Free space in ccp -26%

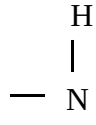
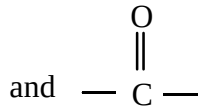
Free space in bcc - 32%

54. The polymer containing strong intermolecular forces e.g, hydrogen bonding is

1) natural rubber 2) teflon 3) nylon 6,6 4) polystyrene

Ans : 3

Sol : Nylon 6,6

Hydrogen bonding between  and  in Nylon -6,6.

55. At 25^0C, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions start precipitating in the form of $Mg(OH)_2$ from a solution of 0.001 M Mg^{2+} ions?

1) 8 2) 9 3) 10 4) 11

Ans : 3

Sol : $k_{sp\left[Mg(OH)_2\right]} = \left[Mg^{2+}\right]\left[OH^-\right]^2$

$\therefore \left[OH^-\right]^2 = \frac{10^{-11}}{10^{-3}} = 10^{-8}$

$\left[OH^-\right] = 10^{-4}$

$pOH = 4$

$pH = 10$

56. The correct order of $E^0{}_{Mg^{2+}/M}$ values with negative sign for the four successive elements Cr, Mn, Fe and Co is

1) Cr > Mn > Fe > Co 2) Mn > Cr > Fe > Co

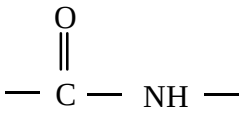
3) Cr > Fe > Mn > Co 4) Fe > Mn > Cr > Co

Ans: 1

57. Biuret test is not given by

1) proteins 2) carbohydrates 3) polypeptides 4) urea

Ans : 2

Sol : biuret test is given by compounds with peptide linkages (). Urea on heating forms peptide linkage. No peptide linkage in carbohydrates

58. The time for half life period of a certain reaction $A \rightarrow$ Products is 1 hour, When the initial concentration of the reactant 'A', is 2.0 mol L^{-1}, how much time does it take for its concentration to come from 0.50 to 0.25 mol L^{-1} if it is a zero order reaction?

1) 1h 2) 4 h 3) 0.5 h 4) 0.25 h

Ans : 4

Sol: for a zero orer reaction $t_{1/2} \alpha a$

2.0 mol L^{-1} $\rightarrow 1.0 mol\ L^{-1}; t^{1/2} = 1\ hour$

0.5 mol L^{-1}

59. A solution containing 2.675 g of $CoCl_3.6NH_3$ (molar mass = 267.5 g mol^{-1}) is passed through a cation exchanger. The chloride ions obtained in solution were treated with excess of $AgNO_3$ to give 4.78 g of AgCl (molar mass = 143.5 g mol^{-1}). The formula of the co-ordination sphere
(At. mass of Ag = 108 u)

1) $\left[CoCl\left(NH_3\right)_5\right]Cl_2$ 2) $\left[Co\left(NH_3\right)_6\right]Cl_3$

3) $\left[CoCl_2\left(NH_3\right)_4\right]Cl$ 4) $\left[CoCl_3\left(NH_3\right)_3\right]$

Ans : 2

Sol : $2.675g\ CoCl_3.6NH_3 \rightarrow 4.78g\ AgCl$
$267.5g\ CoCl_3.6NH_3 \rightarrow 478g\ AgCl$

No of moles AgCl = $\frac{478}{143.5} = 3(approx)$

1mole complex has 3moles of Cl^- ions outside of complex

$\therefore 3Cl^- + 3Ag^+ \rightarrow 3AgCl\downarrow$

60. The standard enthalpy of formation of NH_3 is - 46.0 kJ mol^{-1} . If the enthalpy of formation of H_2 from its atoms is -436 kJ mol^{-1} and that of N_2 is -712 kJ mol^{-1}, the average bond enthalpy of N - H bond in NH_3 is

1) $-1102\ kJ\ mol^{-1}$ 2) $-964\ kJ\ mol^{-1}$ 3) $+352\ kJ\ mol^{-1}$ 4) $+1056\ kJ\ mol^{-1}$

Ans: 3

Sol : $\frac{1}{2}N_2 + \frac{3}{2}H_2 \rightarrow NH_3$

$-46 = \left[\frac{1}{2}\times 712 + \frac{3}{2}\times 436\right] - \left[3\times N - H\right]$

B. E of N-H = +352 KJ

MATHEMATICS

61. Consider the following relations :
R = {(x, y) | x, y are real numbers and x = wy for some rational number w} ;

S = $\{(\frac{m}{n}, \frac{p}{q}) \mid$ m, n, p and q are integers such that $n, q \neq 0$ and qm = pn$\}$. Then

1) R is an equivalence relation but S is not an equivalence relation
2) neither R nor S is an equivalence relation
3) S is an equivalence relation but R is not an equivalence relation
4) R and S both are equivalence relations

Ans : 3

Sol : xRx *for* $w=1 \Rightarrow$ *reflexive, but not symmetric*

R is not equivalence relation

$\frac{m}{n} S \frac{m}{n} \Rightarrow$ *reflexive*

$\frac{m}{n} S \frac{p}{q}, \frac{p}{q} S \frac{r}{s}$

$qm = pn, ps = qr$

62. The number of complex numbers z such that $|z-1| = |z+1| = |z-i|$ equals

1) 0 2) 1 3) 2 4) ∞

Ans : 2

Sol : Z is the circumcentre of the triangle formed by A (`1,0), B(-1,0), C(0,1) $\Rightarrow$ only one point

63. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$

1) -2 2) -1 3) 1 4) 2

Ans : 3

Sol : $x = \frac{1 \pm i\sqrt{3}}{2}$

$\Rightarrow \alpha = -\omega, \beta = -\omega^2$

$\alpha^{2009} + \beta^{2009} = 1$

64. Consider the system of linear equations :

$$x_1 + 2x_2 + x_3 = 3$$
$$2x_1 + 3x_2 + x_3 = 3$$
$$3x_1 + 5x_2 + 2x_3 = 1$$

The system has

1) infinite number of solutions 2) exactly 3 solutions

3) a unique solution 4) no solution

Ans : 4

Sol : $(1)-(2) \Rightarrow x_1 + x_2 = 0$

$(2)-3(1) \Rightarrow x_1 + x_2 = 2$

$N0$ *sol*

65. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is

1) 3 2) 36 3) 66 4) 108

Ans : 4

Sol : $3_{C_2} \times 9_{C_2} = 108$

66. Let $f : (-1,1) \to R$ be a differentiable function with f(0) = -1 and f'(0) = 1. Let $g(x) = [f(2f(x)+2)]^2$. Then $g'(0) =$

1) 4 2) -4 3) 0 4) -2

Ans : 2

Sol : $g(x)=[f(2f(x)+2)]^2$

$g'(x)=2[f(2f(x)+2)].f'[2f(x)+2].2f'(x)$

$g'(0)=2[f(0)].f'(0).2f'(0)=-4$

67. Let $f:R\to R$ be a positive increasing function with $\lim\limits_{x\to\infty}\frac{f(3x)}{f(x)}=1.$ Then $\lim\limits_{x\to\infty}\frac{f(2x)}{f(x)}=$

1) 1 2) $\frac{2}{3}$ 3) $\frac{3}{2}$ 4) 3

Ans : 1

Sol : $x\le 2x\le 3x$

$\Rightarrow f(x)\le f(2x)\le f(3x)$

$\Rightarrow \underset{x\to\infty}{Lt}1\le\frac{f(2x)}{f(x)}\le\frac{f(3x)}{f(x)}$

$\underset{x\to\infty}{Lt}\frac{f(2x)}{f(x)}=1$

68. Let p(x) be a function defined on **R** such that $p'(x)=p'(1-x),\ \text{for all } x\in[0,1],\ p(0)=1$ and $p(1)=41$. Then $\int_0^1 p(x)\,dx$ equals

1) $\sqrt{41}$ 2) 21 3) 41 4) 42

Ans : 2

Sol : $p'(x)=p'(1-x),\forall x\in[0,1],p(0)=1,p(1)=41$

$\Rightarrow\int p'(x)dx=\int p'(1-x)dx+c$

$\Rightarrow p(x)=-p(1-x)+c$

$\Rightarrow c=p(1)+p(0)=42$

$\therefore p(x)+p(1-x)=42,\forall x\in[0,1]$

$I=\int_0^1 p(x)dx=\int_0^1 p(1-x)dx$

$\therefore 2I=\int_0^1[p(x)+p'(x)]dx=\int_0^1 42dx=42$

69. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1=a_2=.....=a_{10}=150$ and $a_{10},a_{11},....$ are in an AP with common difference -2, then the time taken by him to count all notes is

1) 24 minutes 2) 34 minutes 3) 125 minutes 4) 135 minutes

Ans : 2

Sol : First ten minutes = 10 x 150 = 1500 remaining = 3000

a_{11} = 148,a_{12} = 146,----

$3000 = \frac{n}{2}\{2\times148 + (n-1)(-2)\}$

$\Rightarrow n = 24$

$total = 10 + 24 = 34$

70. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is

1) y = 0 2) y = 1 3) y = 2 4) y = 3

Ans : 4

Sol : $\frac{dy}{dx} = 1 - \frac{8}{x^3} = 0$

$\Rightarrow x = 2, y = 3$

$\therefore Tangent\ equation\ is\ y = 3$

71. The area bounded by the curve y = cos x and y = sin x between the ordinates x = 0 and $x = \frac{3\pi}{2}$ is

1) $4\sqrt{2} - 2$ 2) $4\sqrt{2} + 2$ 3) $4\sqrt{2} - 1$ 4) $4\sqrt{2} + 1$

Ans : 1

Sol : $Area = \int_0^{\pi/4} (\cos x - \sin x)dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)dx + \int_{5\pi/4}^{3\pi/2} (\cos x - \sin x)dx$

$= \left(\sqrt{2} - 1\right) + 2\sqrt{2} + \left(\sqrt{2} - 1\right)$

$= \left(4\sqrt{2} - 2\right)$

72. Solution of the differential equation $\cos x\, dy = y(\sin x - y)dx,\ 0 < x < \frac{\pi}{2}$ is

1) $\sec x = (\tan x + c)y$ 2) $y \sec x = \tan x + c$

3) $y \tan x = \sec x + c$ 4) $\tan x = (\sec x + c)y$

Ans : 1

Sol : $\cos x\frac{dy}{dx} = y\sin x - y^2$

$\frac{dy}{dx} - y\tan x = -y^2\sec x$

$\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\tan x = -\sec x$

$Let\ z = \frac{-1}{y} \Rightarrow \frac{dz}{dx} = \frac{1}{y^2}\frac{dy}{dx}$

$\frac{dz}{dx} + z\tan x = -\sec x$

$I.F = e^{\int \tan x dx} = \sec x$

$sol.is\ z.\sec x = \int \sec x(-\sec x)dx + c$

$\sec x = y(\tan x + c)$

73. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a}.\vec{b} = 3$ is

1) $-\hat{i} + \hat{j} - 2\hat{k}$ 2) $2\hat{i} - \hat{j} + 2\hat{k}$ 3) $\hat{i} - \hat{j} - 2\hat{k}$ 4) $\hat{i} + \hat{j} - 2\hat{k}$

Ans : 1

Sol : $\bar{a} \times \bar{b} = -\bar{c}$

$take \times with\ \bar{a}$

$\bar{a} \times \bar{a} \times \bar{b} = \bar{c} \times \bar{a}$

$3a - 2b = 2i + j + k$

$\therefore b = -i + j - 2k$

74. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$

1) (-3, 2) 2)(2, -3) 3) (-2, 3) 4)(3, -2)

Ans : 1

Sol : $a.c = 0 \Rightarrow \lambda - 1 + 2\mu = 0$

$b.c = 0 \Rightarrow 2\lambda + 4 + \mu = 0$

$\therefore (\lambda, \mu) = (-3, 2)$

75 If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

1) $x = 1$ 2) $2x + 1 = 0$ 3) $x = -1$ 4) $2x - 1 = 0$

Ans : 3

Sol : Drirectrix is x = -1

76. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is

1) $\frac{23}{\sqrt{15}}$ 2) $\sqrt{17}$ 3) $\frac{17}{\sqrt{15}}$ 4) $\frac{23}{\sqrt{17}}$

Ans : 4

Sol : $\frac{13}{5}+\frac{32}{b}=1\Rightarrow b=-20$

$L\ is\ 4x-y-20=0$

$k\ is\ \ parallel\ to\ L\Rightarrow c=\frac{3}{4}$

$K\ is\ 4x-y+3=0$

$d=\frac{23}{\sqrt{17}}$

77. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an angle θ with the positive z-axis, then θ equals

1) $30°$ 2) $45°$ 3) $60°$ 4) $75°$

Ans : 3

Sol : $\cos^2 45+\cos^2 120+\cos^2\theta=1$

$\Rightarrow \theta=60^0$

78. Let S be a non-empty subset of **R**. Consider the following statement :
P : There is a rational number $x\in S$ such that x > 0.
Which of the following statements is the negation of the statement P ?

1) There is a rational number $x\in S$ such that $x\le 0$.

2)There is no rational number $x\in S$ such that $x\le 0$.

3) Every rational number $x\in S$ satisfies $x\le 0$.

4) $x\in S$ and $x\le 0\Rightarrow$ x is not rational.

Ans : 1

Sol : negation is there is a rational number $x\in S\ such\ that\ x\le 0$

79. Let $\cos(\alpha+\beta)=\frac{4}{5}$ and let $\sin(\alpha-\beta)=\frac{5}{13}$, where $0\le\alpha,\beta\le\frac{\pi}{4}$. Then $\tan 2\alpha=$

1) $\frac{25}{16}$ 2) $\frac{56}{33}$ 3) $\frac{19}{12}$ 4) $\frac{20}{7}$

Ans : 2

Sol : $\tan 2\alpha=\tan\left((\alpha+\beta)+(\alpha-\beta)\right)$

80. The circle $x^2+y^2=4x+8y+5$ intersects the line 3x - 4y = m at two distinct points if

1) $-85<m<-35$ 2) $-35<m<15$ 3) $15<m<65$ 4) $35<m<85$

Ans : 2

Sol : $c(2,4), r=5$

$$d=\frac{|3.2-4.4-m|}{\sqrt{9+16}}=\frac{|m+10|}{5}$$

$$d<r \Rightarrow \frac{|m+10|}{5}<5$$

$$\Rightarrow -35<m<15$$

81. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

1) $\frac{5}{2}$ 2) $\frac{11}{2}$ 3) 6 4) $\frac{13}{2}$

Ans : 2

$$n_1=n_2=5, \mu_1=2, \mu_2=4, {\sigma_1}^2=4, {\sigma_2}^2=5$$

$$\sigma^2=\frac{1}{n_1+n_2}\left(n_1{\sigma_1}^2+n_2{\sigma_2}^2+\frac{n_1n_2}{n_1+n_2}(\mu_1-\mu_2)^2\right)$$

Sol : $=\frac{1}{10}\left(20+25+\frac{25}{10}(4)\right)$

$=\frac{11}{2}$

82. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is

1) $\frac{1}{3}$ 2) $\frac{2}{7}$ 3) $\frac{1}{21}$ 4) $\frac{2}{23}$

Ans : 2

Sol : Required probability = $\frac{4.3.2}{9_{C_3}}$

83. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is

1) There is a regular polygon with $\frac{r}{R}=\frac{1}{2}$ 2)There is a regular polygon with $\frac{r}{R}=\frac{1}{\sqrt{2}}$

3) There is a regular polygon with $\frac{r}{R}=\frac{2}{3}$ 4) There is a regular polygon with $\frac{r}{R}=\frac{\sqrt{3}}{2}$

Ans : 3

Sol : $\frac{r}{R}=\sin\frac{\pi}{n}, n=no.of\ sides$

For no integral value of n, $\frac{r}{R}=\frac{2}{3}$

84. The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0, is

1) less than 4 2) 5 3)6 4) at least 7

Ans : 4

85. Let $f : R \to R$ be defined by $f(x) = \begin{cases} k - 2x, & \text{if } x \le -1 \\ 2x + 3, & \text{if } x > -1 \end{cases}$. If f has a local minimum at x = -1, then a possible value of k is

1) 1 2) 0 3) $\frac{-1}{2}$ 4) -1

Ans : 4

Sol : Minimum value exists at f(-1)

k + 2 = -2 + 3

Directions : Questions number 86 to 90 are Assertion - Reason type questions. Each of these questions contains two statements.

Statement-1 : (Assertion) and Statement-2 : (Reason).

Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

86. Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ... , 20}.

Statement-1 : The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement-2 : If the four chosen numbers form an AP, then the set of all possible values of common difference is $\{\pm1, \pm2, \pm3, \pm4, \pm5\}$.

1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

3) Statement-1 is true, Statement-2 is false.

4) Statement-1 is false, Statement-2 is true.

Ans : 3

Sol : Let the number be $n(E) = 17 + 14 + 11 + 8 + 5 + 2,$

$n(S) = 20_{C_4}$

St.1 is correct

St.2 is wrong

87. Let $S_1 = \sum_{j=1}^{10} j(j-1)10C_j$, $S_2 = \sum_{j=1}^{10} j10C_j$ and $S_3 = \sum_{j=1}^{10} j^2 10C_j$.

Statement-1 : $S_3 = 55 \times 2^9$.

Statement-2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

3) Statement-1 is true, Statement-2 is false.

4) Statement-1 is false, Statement-2 is true.

Ans : 3

Sol : $S_1 = \sum_{j=1}^{10} j(j-1)10_{C_j} = \sum_{j=2}^{10} j(j-1)\frac{10}{j}.\frac{9}{j-1}.8_{C_{(l-2)}}$

$= \sum_{j=2}^{10} 90.\left(8_{C_0} + ... + 8_{C_0}\right) = 90.2^8$

$S_2 = \sum_{1}^{10} j.\frac{10}{j}9_{C_{(j-1)}} = 10.2^9$

$S_3 = \sum\left(j(j-1)+j\right)10_{C_j} = S_1 + S_2 = 90.2^8 + 10.2^9 = 2^9\left(45+10\right) = 55.2^9$

88. Statement-1 : The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane x - y + z = 5.

Statement-2 : The plane x - y + z = 5 bisects the line segment joining A(3, 1, 6) and B(1,3,4)

1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

3) Statement-1 is true, Statement-2 is false.

4) Statement-1 is false, Statement-2 is true.

Ans : 2

Sol : Image of A (3,1,6) w.r.t $x - y + 3 - 5 = 0$ *is* (h,k,l)

$\Rightarrow \frac{h-3}{1} = \frac{k-1}{-1} = \frac{l-6}{1} = \frac{-2\left(3-1+6-5\right)}{1+1+1} = -2$

$h = -2+3 = 1, k = 2+1 = 3, l = -2+6 = 4$

Image = B(1,3,4)

89. Let $f : R \to R$ be a continuous function defined by $f(x) = \frac{1}{e^x + 2e^{-x}}$.

Statement-1 : $f(c) = \frac{1}{3}$, for some $c \in R$.

Statement-2 : $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all $x \in R$.

1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

3) Statement-1 is true, Statement-2 is false.

4) Statement-1 is false, Statement-2 is true.

Ans : 1

Sol : $e^x and e^{-x}$ are positive values

$AM \geq GM \Rightarrow \frac{e^x + 2e^{-x}}{2} \geq \sqrt{e^x.2e^{-x}} = \sqrt{2}$

$e^x + 2e^{-x} \geq 2\sqrt{2}$ *so* $\frac{1}{e^x + 2x^{-x}} \leq \frac{1}{2\sqrt{2}}$

$\therefore 0 < f(x) \leq \frac{1}{2\sqrt{2}}$ *for all* $x \in R$.

since $\frac{1}{3} \in \left(0, \frac{1}{2\sqrt{2}}\right)$ *and* f *is continuous, for some* $C \in R$, *he get* $f(e) = \frac{1}{3}$

90 Let A be a 2×2 matrix with non-zero entries and let $A^2 = I.$, where I is 2×2 identity matrix.
Define Tr(A) = sum of diagonal elements of A and |A| = determinant of matrix A.
Statement-1 : Tr(A) = 0.
Statement-2 : |A| = 1.
1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
3) Statement-1 is true, Statement-2 is false.
4) Statement-1 is false, Statement-2 is true.
Ans : 3

Sol :

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $A^2 \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = I$

$\therefore a^2 + bc = 1 = bc + d^2 and$

$ab + bd = ac + cd = 0 \Rightarrow d = -a$ since b ≠ 0,c ≠ 0

$\therefore$ A$= \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ *has* $T_r A = 0$ *and* $|A| = -a^2 - bc = -\left(a^2 + bc\right) = -1$

Statemet I is true, II is false.