AIEEE 2010

25'Th April, 2010 | 9:30 AM – 12:30 PM

Code C

Maths

1. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is

   (1) $\frac{23}{\sqrt{17}}$

   (2) $\frac{23}{\sqrt{15}}$

   (3) $\sqrt{17}$

   (4) $\frac{17}{\sqrt{15}}$

Solution:

(1) $\frac{x}{5} + \frac{y}{b} = 1$ passes through (13, 32)

$\Rightarrow \frac{13}{5} + \frac{32}{b} = 1 \Rightarrow b = -20$

$\Rightarrow \frac{x}{5} - \frac{y}{20} = 1$ is the line L

$= 4x - y = 20$

slope $= 4$

$\frac{x}{c} + \frac{y}{3} = 1$ is $\parallel$ to L

slope $= -\frac{3}{c} = 4 \Rightarrow c = -\frac{3}{4}$

Eqn. of line K is: $-\frac{4x}{3} + \frac{y}{3} = 1$

$4x - y = -3$

Distance $= \frac{|20 - (-3)|}{\sqrt{4^2 + 1^2}} = \frac{23}{\sqrt{17}}$

2. The number of 3 × 3 non-singular matrices, with four entries as 1 and all other entries as 0, is

(1) at least 7

(2) less than 4

(3) 5

(4) 6

Solution:

(2) 4 enteries 1 and 5 entries zero

Cases for 5 zeroes

	R_1	R_2	R_3	
Case I	2	2	1	→ 27 determinants
" II	1	2	2	→ " "
" III	2	1	2	→ 27 "

For each case u will have same answer (Reason being only rows get interchanged)

Aim for 27 determinants & check for non-singular

$$\begin{vmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} \quad \begin{vmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix} \quad \begin{vmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

Singular Singular Singular

$$\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} \quad \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} \quad \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{vmatrix}$$

Singular Non singular Non-singular

$$\begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{vmatrix} \quad \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{vmatrix} \quad \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix}$$

Non-singular singular Non-singular

No need to proceed further as in case I (when only nine were looked) 4 were non-singular. (u have to tell atleast only)

Similar 4 & 4 will occur for case II & case III.

So, "atleast 7" is correct option

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3. Let $f : \mathbf{R} \to \mathbf{R}$ be defined by

$$f(x) = \begin{cases} k - 2x, & \text{if } x \le -1 \\ 2x + 3, & \text{if } x > -1 \end{cases}$$

If f has a local minimum at $x = -1$ then a possible value of k is

(1) -1

(2) 1

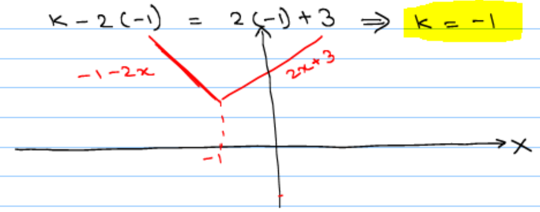
(3) 0

(4) $-\frac{1}{2}$

Solution:

3) f^n must be continuous at $x = -1$

$k - 2(-1) = 2(-1) + 3 \Rightarrow \boxed{k = -1}$



Directions : Questions number 4 to 8 are Assertion – Reason type questions. Each of these questions contains two statements.

Statement-1 : (Assertion) and

Statement-2 : (Reason).

Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

4. Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ..., 20}.

Statement-1 : The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement-2 : If the four chosen numbers form an AP, then the set of all possible values of common difference is {±1, ±2, ±3, ±4, ±5}.

(1) Statement-1 is false, Statement-2 is true.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1.

(4) Statement-1 is true, Statement-2 is false.

Solution:

4) common diff. 1 : (CD 1)

(1,2,3,4) - - - - - - - (17,18,19,20)

CD 2

(1,3,5,7) ; (2,4,6,8) - - - - - (14,16,18,20)

CD 3

(1,4,7,10) - - - - - - - (11,14,17,20)

CD 4

(1,5,9,13) - - - - - - - (8,12,16,20)

CD 5

(1,6,11,16) - - - - - - (5,10,15,20)

CD 6

(1,7,13,19) , (2,8,14,20)

straight away statement two is false

we do not need to check for statement 1 but still....

CD-1 ⇒ 17

CD-2 ⇒ 14

CD-3 ⇒ 11

CD-4 ⇒ 8

CD-5 ⇒ 5

CD-6 ⇒ 2

Total = 57

Hence probability $= \frac{57}{{}^{20}C_4} = \frac{1}{85}$

5. Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. Define
Tr(A) = sum of diagonal elements of A and
| A | = determinant of matrix A.

Statement-1 : $Tr(A) = 0$.

Statement-2 : $|A| = 1$.

(1) Statement-1 is false, Statement-2 is true.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1.

(4) Statement-1 is true, Statement-2 is false.

Solution:

5)

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2+bc & (a+d)b \\ c(a+d) & bc+d^2 \end{bmatrix}$$

$$A^2 = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a^2+bc & (a+d)b \\ c(a+d) & (bc+d^2) \end{bmatrix}$$

$b(a+d) = 0$ & $c(a+d) = 0$

As $b, c \neq 0 \Rightarrow a+d = 0$
$\Rightarrow a = -d$

& $a^2 + bc = 1$

$|A| = ad - bc = a(-a) - bc$
$= -(a^2 + bc) = -1$

Hence $Tr(A) = 0$
but $|A| \neq -1$

St-1 is true & St-2 is false

6. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function defined by

$$f(x) = \frac{1}{e^x + 2e^{-x}}.$$

Statement-1 : $f(c) = \frac{1}{3}$, for some $c \in \mathbb{R}$.

Statement-2 : $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all $x \in \mathbb{R}$.

(1) Statement-1 is false, Statement-2 is true.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is true; Statement-2 is *not* a correct explanation for Statement-1.

(4) Statement-1 is true, Statement-2 is false.

Solution:

6) $f(x) = \frac{1}{e^x + 2e^{-x}}$

$f(c) = \frac{1}{3} \Rightarrow e^c + 2e^{-c} = 3$

Let $e^c = t$

$t + \frac{2}{t} = 3 \Rightarrow t^2 - 3t + 2 = 0$

$\Rightarrow t = 1$ or $t = 2$

$e^c = 1$ $\quad$ $e^c = 2$

$\Rightarrow c = 0$ $\quad$ $c = \ln 2$

Hence statement-I is true.

$t = e^x > 0$

$y = \frac{1}{t + \frac{2}{t}} = \frac{t}{t^2+2} > 0$

$t^2y - t + 2y = 0$

$D = 1 - 8y^2 \geq 0 \Rightarrow 8y^2 - 1 \le 0$

$\left(y - \frac{1}{2\sqrt{2}}\right)\left(y + \frac{1}{2\sqrt{2}}\right) \le 0$ → always +ve

$y - \frac{1}{2\sqrt{2}} \le 0 \Rightarrow y \le \frac{1}{2\sqrt{2}}$

Hence $y \in \left(0, \frac{1}{2\sqrt{2}}\right)$

$y = \frac{1}{3}$ lies in $y \in \left(0, \frac{1}{2\sqrt{2}}\right)$

Hence statement-II explains statement I

7. **Statement-1 :** The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane $x - y + z = 5$.

Statement-2 : The plane $x - y + z = 5$ bisects the line segment joining A(3, 1, 6) and B(1, 3, 4).

(1) Statement-1 is false, Statement-2 is true.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is true; Statement-2 is *not* a correct explanation for Statement-1.

(4) Statement-1 is true, Statement-2 is false.

Solution:

7) use the result:

$$\frac{h-x_1}{A} = \frac{k-y_1}{B} = \frac{l-z_1}{C} = \frac{-2(Ax_1+By_1+Cz_1+D)}{A^2+B^2+C^2}$$

Plane: $Ax+By+Cz+D=0$
object $(3,1,6)$
Image $(1,3,4)$

$$\frac{h-3}{1} = \frac{k-1}{-1} = \frac{l-6}{1} = \frac{-2(3-1+6-5)}{1^2+(-1)^2+1^2}$$

$$h-3 = \frac{k-1}{-1} = l-6 = -2$$

$h=1$, $k=3$, $l=4$

st-1 is correct.

Statement-2: $(x_m, y_m, z_m) = \left(\frac{x_A+x_B}{2}, \ldots\ldots, \frac{z_A+z_B}{2}\right)$

Mid point $= \left(\frac{3+1}{2}, \frac{1+3}{2}, \frac{6+4}{2}\right)$

$\equiv (2, 2, 5)$

$x-y+z=5$ satisfies the eqn. of plane

If you know, above result is derived by the use of statement-2 (i.e. mid-point of A, B lies on the plane.

Hence st-1 is correct & st-2 explains st-1.

OR: mid-point of A & B lies on plane & direction cosines of AB = direction " " normal of the plane. [Infact these two conditions are used to find above result]

8. Let $S_1 = \sum_{j=1}^{10} j(j-1)\, {}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\, {}^{10}C_j$

and $S_3 = \sum_{j=1}^{10} j^2\, {}^{10}C_j$.

Statement-1 : $S_3 = 55 \times 2^9$.

Statement-2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(1) Statement-1 is false, Statement-2 is true.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is true; Statement-2 is *not* a correct explanation for Statement-1.

(4) Statement-1 is true, Statement-2 is false.

Solution:

8) $S_1 = \sum_{j=1}^{10} j(j-1)\, {}^{10}C_j = 10 \times 9 \sum_{j=2}^{10} {}^{8}C_{j-2}$

$S_1 = 10 \times 9 \left[2^8\right]$

$S_2 = \sum_{j=1}^{10} j\, {}^{10}C_j = 10 \times 2^9$

$\left[\sum_{r=0/1}^{n} r\, {}^{n}C_r = n\, 2^{n-1}\right]$

$S_3 = \sum_{j=1}^{10} j^2\, {}^{10}C_j = (10)(11) \times 2^8$

$= 55 \times 2^9$

Statement-I is True

Statement-II is false as S_1 is correct but S_2 is false.

Hence (D) is correct

9. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals

(1) 75°

(2) 30°

(3) 45°

(4) 60°

Solution:

9) $l = \cos 45^\circ$, $m = \cos 120^\circ = -\frac{1}{2}$

$l^2 + m^2 + n^2 = 1$

$\frac{1}{2} + \left(\frac{1}{4}\right) + n^2 = 1 \Rightarrow n^2 = \frac{1}{4}$

$n = \frac{1}{2}$; $n = -\frac{1}{2}$

$\hookrightarrow$ not possible (AB makes acute ∠)

$\cos\gamma = \frac{1}{2}$

$\gamma = 60^\circ$

10. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A ***false*** statement among the following is

(1) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

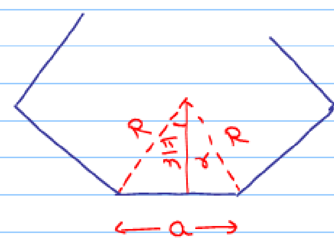
(2) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

(3) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(4) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

Solution:

10)



$R\cos\frac{\pi}{n} = r$

$\frac{r}{R} = \cos\frac{\pi}{n}$

$\frac{r}{R} = \cos\frac{\pi}{n} = \frac{\sqrt{3}}{2}$ for $n = 6$ $\Rightarrow$ True

$\frac{r}{R} = \cos\frac{\pi}{n} = \frac{1}{2}$ for $n = 3$ $\Rightarrow$ True

$\frac{r}{R} = \cos\frac{\pi}{n} = \frac{1}{\sqrt{2}}$ for $n = 4$ $\Rightarrow$ True

$\cos\frac{\pi}{n} = \frac{2}{3}$ has no integral value of n.

11. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and

let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$.

Then $\tan 2\alpha =$

(1) $\frac{20}{7}$ (2) $\frac{25}{16}$

(3) $\frac{56}{33}$ (4) $\frac{19}{12}$

Solution:

11) $\cos(\alpha+\beta) = \frac{4}{5}$, $\sin(\alpha-\beta) = \frac{5}{13}$

$\Downarrow$ $\Downarrow$

$\tan(\alpha+\beta) = \frac{3}{4}$ $\tan(\alpha-\beta) = \frac{5}{12}$

$$\tan(\overline{\alpha+\beta} + \overline{\alpha-\beta}) = \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \times \frac{5}{12}} = \frac{56}{33}$$

12. Let S be a non-empty subset of **R**. Consider the following statement :

P : There is a rational number $x \in S$ such that $x > 0$.

Which of the following statements is the negation of the statement P ?

(1) $x \in S$ and $x \le 0 \Rightarrow x$ is not rational.

(2) There is a rational number $x \in S$ such that $x \le 0$.

(3) There is no rational number $x \in S$ such that $x \le 0$.

(4) Every rational number $x \in S$ satisfies $x \le 0$.

Solution:

13. Let p(x) be a function defined on $\mathbb{R}$ such that $p'(x) = p'(1-x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)\,dx$ equals

(1) 42

(2) $\sqrt{41}$

(3) 21

(4) 41

Solution:

(13) $p'(x) = p'(1-x)$

$\Rightarrow p'(x) - p'(1-x) = 0$

Integrate to get

$p(x) + p(1-x) = K$

Substitute $x = 1$

$p(1) + p(1-1) = K$

$41 + 1 = K \Rightarrow K = 42$

$I = \int_0^1 p(x)\,dx$ — (i)

$x = 1-t \Rightarrow dx = -dt$

$I = \int_1^0 p(1-t)(-dt)$

$I = \int_0^1 p(1-t)\,dt$ — (ii)

or use a−x property

Operate (i) + (ii)

$2I = \int_0^1 (p(x) + p(1-x))\,dx$

$2I = \int_0^1 42\,dx = 42$

$I = 21$

14. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \ldots = a_{10} = 150$ and $a_{10}, a_{11}, \ldots$ are in an AP with common difference -2, then the time taken by him to count all notes is

(1) 135 minutes

(2) 24 minutes

(3) 34 minutes

(4) 125 minutes

Solution:

14) till 9 th trial, he counts 1350 notes

notes left = 3150

$$3150 = \frac{n'}{2}\left[2\times150 + (n'-1)(-2)\right]$$

$$3150 = n'\left[151 - n'\right]$$

$$\Rightarrow n'^2 - 151n' + 3150 = 0$$

$$\Rightarrow n' = 25,\ 126$$

rejected

Hence total minutes = $9 + 25 = 34$

15. Let $f : \mathbf{R} \to \mathbf{R}$ be a positive increasing function with $\lim\limits_{x\to\infty} \frac{f(3x)}{f(x)} = 1$.

Then $\lim\limits_{x\to\infty} \frac{f(2x)}{f(x)} =$

(1) 3

(2) 1

(3) $\frac{2}{3}$

(4) $\frac{3}{2}$

Solution:

15. $x < 2x < 3x$

As $f(x)$ is Inc

$$f(x) < f(2x) < f(3x)$$

As $f(x)$ is positive, divide by $f(x)$

$$1 < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}$$

Take $\lim_{x\to\infty}$ $\quad \lim_{x\to\infty} 1 < \lim_{x\to\infty}\frac{f(2x)}{f(x)} < \lim_{x\to\infty}\frac{f(3x)}{f(x)}$

Using Sandwitch Thrm, $\lim_{x\to\infty}\frac{f(2x)}{f(x)} = 1$ $\quad$ (2)

16. Solution of the differential equation

$\cos x\, dy = y(\sin x - y)\, dx,\ 0 < x < \frac{\pi}{2}$ is

(1) $\tan x = (\sec x + c)\, y$

(2) $\sec x = (\tan x + c)\, y$

(3) $y \sec x = \tan x + c$

(4) $y \tan x = \sec x + c$

Solution:

16) $\cos x\, dy - y \sin x\, dx + y^2 dx = 0$

$$\frac{dy}{dx} - \tan x\, y = -y^2 \sec x$$

It is of the form

$$\frac{dy}{dx} + Py = Qy^n$$

where P, Q are constt. or f^n of x.

& sol^n is $y^{1-n}(I.F) = \int (1-n)\, Q\, (I.F)\, dx + C$

where $I.F. = e^{\int (1-n) P dx}$

$$I.F. = e^{\int (1-2)(-\tan x) dx} = e^{\ln \sec x} = \sec x$$

$$y^{1-2}(\sec x) = \int (1-2) \sec x (\sec x)\, dx + C$$

$$y^{-1} \sec x = \tan x + C$$

$$\sec x = (\tan x + c)\, y$$

17. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is

(1) $4\sqrt{2} + 1$

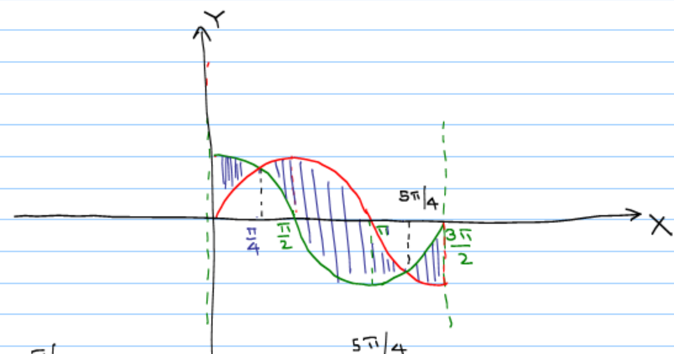
(2) $4\sqrt{2} - 2$

(3) $4\sqrt{2} + 2$

(4) $4\sqrt{2} - 1$

Solution:

17)



$$\int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\,dx$$

$$+ \int_{5\pi/4}^{3\pi/2} (\cos x - \sin x)\,dx$$

$$\Big[\sin x + \cos x\Big]_0^{\pi/4} + \Big[-\cos x - \sin x\Big]_{\pi/4}^{5\pi/4} + \Big(\sin x + \cos x\Big)\Big|_{5\pi/4}^{3\pi/2}$$

$$(\sqrt{2} - 1) + (-1)\left[(-\sqrt{2}) - (-\sqrt{2})\right] + (-1 + \sqrt{2})$$

$$(\sqrt{2} - 1) + (2\sqrt{2}) + (\sqrt{2} - 1)$$

$$4\sqrt{2} - 2$$

18. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is

(1) $y = 3$

(2) $y = 0$

(3) $y = 1$

(4) $y = 2$

Solution:

18) $y = x + \frac{4}{x^2}$

$\frac{dy}{dx} = 1 - \frac{8}{x^3} = 0$ ($\because$ line is $\parallel$ to x-axis)

$x = 2 \Rightarrow y = 2 + \frac{4}{2^2} = 3$

Eqn. of tangent is:

$\frac{y-3}{x-2} = 0 \Rightarrow y = 3$

19. Let $f : (-1, 1) \to \mathbb{R}$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$

(1) -2

(2) 4

(3) -4

(4) 0

Solution:

19) $f : (-1,1) \longrightarrow R$, $f(0) = -1$, $f'(0) = 1$

$g(x) = [f(2f(x) + 2)]^2$

$g'(x) = 2[f(2f(x)+2)]\,[f'(2f(x)+2)\,(2f'(x))]$

$g'(0) = 2[f(2f(0)+2)]\,[f'(2f(0)+2)][2f'(0)]$

$g'(0) = 2[f(-2+2)]\,[f'(-2+2)]\,2f'(0)$

$= 2f(0)\,f'(0)\,2f'(0)$

$= 2(-1)(1)2(1) = -4$

$\Rightarrow g'(0) = -4$

20. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is

(1) 108

(2) 3

(3) 36

(4) 66

Solution:

20) Urn A: R_1, R_2, R_3

Urn B: $B_1, B_2 \ldots \ldots B_9$

from urn-A, select 2 red in ${}^3C_2$ ways

From urn-B, select 2 blue balls in ${}^9C_2$ ways

No. of ways = ${}^3C_2 \times {}^9C_2$

$3 \times 36 = 108$

21. Consider the system of linear equations :

$$x_1 + 2x_2 + x_3 = 3$$
$$2x_1 + 3x_2 + x_3 = 3$$
$$3x_1 + 5x_2 + 2x_3 = 1$$

The system has

(1) no solution

(2) infinite number of solutions

(3) exactly 3 solutions

(4) a unique solution

Solution:

21) $\overset{A}{\begin{bmatrix}1 & 2 & 1\\ 2 & 3 & 1\\ 3 & 5 & 2\end{bmatrix}}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \overset{B}{\begin{bmatrix}3\\ 3\\ 1\end{bmatrix}}$

$(A|B)$ Augmented matrix $= \left|\begin{array}{ccc:c}1 & 2 & 1 & 3\\ 2 & 3 & 1 & 3\\ 3 & 5 & 2 & 1\end{array}\right|$

$R_2 \to R_2 - 2R_1$ & $R_3 \to R_3 - 3R_1$

$\left|\begin{array}{ccc:c}1 & 2 & 1 & 3\\ 0 & -1 & -1 & -3\\ 0 & -1 & -1 & -8\end{array}\right|$

$R_3 \to R_3 - R_2$

$= \left|\begin{array}{ccc:c}1 & 2 & 1 & 3\\ 0 & -1 & -1 & -3\\ 0 & 0 & 0 & -5\end{array}\right|$

Rank $(A) = 2$, Rank $(A|B) = 3$

Rank $(A) \neq$ Rank $(A|B)$

Hence No solution

22. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a} \cdot \vec{b} = 3$ is

(1) $\hat{i} + \hat{j} - 2\hat{k}$

(2) $-\hat{i} + \hat{j} - 2\hat{k}$

(3) $2\hat{i} - \hat{j} + 2\hat{k}$

(4) $\hat{i} - \hat{j} - 2\hat{k}$

Solution:

22) $\bar{a} \times \bar{b} + \bar{c} = \bar{0}$

Take cross with $\bar{a}$

$\bar{a} \times (\bar{a} \times \bar{b}) + \bar{a} \times \bar{c} = \bar{a} \times \bar{0}$

$(\bar{a} \cdot \bar{b})\,\bar{a} - (\bar{a} \cdot \bar{a})\,\bar{b} + \bar{a} \times \bar{c} = \bar{0}$

where $\bar{a} \times \bar{c} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & -1\\ 1 & -1 & -1\end{vmatrix}$

$-2\hat{i} - \hat{j} - \hat{k}$

$3(\hat{j} - \hat{k}) - (2)\,\bar{b} + (-2\hat{i} - \hat{j} - \hat{k}) = \bar{0}$

$\Rightarrow \bar{b} = -\hat{i} + \hat{j} - 2\hat{k}$

23. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

(1) $\frac{13}{2}$

(2) $\frac{5}{2}$

(3) $\frac{11}{2}$

(4) 6

Solution:

24. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

(1) $35 < m < 85$

(2) $-85 < m < -35$

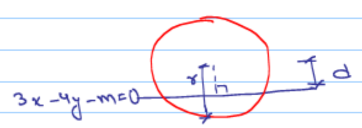
(3) $-35 < m < 15$

(4) $15 < m < 65$

Solution:

(24) $x^2 + y^2 - 4x - 8y - 5 = 0$

$C \equiv (2, 4)$, radis $= \sqrt{4+16+5} = 5$



$d < r$

$$\frac{|3\times2 - 4\times4 - m|}{\sqrt{3^2+4^2}} < 5$$

$|m+10| < 25$

$-25 < m+10 < 25$

$-35 < m < 15$

25. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is

(1) $\frac{2}{23}$

(2) $\frac{1}{3}$

(3) $\frac{2}{7}$

(4) $\frac{1}{21}$

Solution:

(25) $R_1, R_2, R_3,$
B_1, B_2, B_3, B_4
G_1, G_2

Event space: ${}^3C_1 \times {}^4C_1 \times {}^2C_1$

Sample space: ${}^9C_3$

Prob. $= \frac{3\times4\times2}{\frac{9\times8\times7}{6}} = \frac{6\times3\times4\times2}{9\times8\times7}$

$= \frac{2}{7}$

26. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

(1) $2x - 1 = 0$

(2) $x = 1$

(3) $2x + 1 = 0$

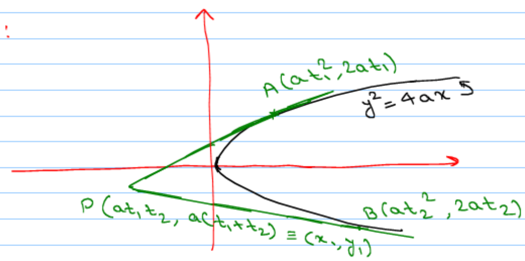
(4) $x = -1$

Solution:

(26) As a result we know locus of point of intersection of $\perp r$ tangent is the directrix of the parabola $y^2 = 4ax$

hence $x = -a$

Proof:



Slope of tangents at A & B are $\frac{1}{t_1}$ & $\frac{1}{t_2}$.

prod. of slopes $= \frac{1}{t_1}\frac{1}{t_2} = -1$

[$\because$ lines are $\perp r$]

$\Rightarrow t_1 t_2 = -1$

But $x = at_1t_2$

$\Rightarrow x = -a$

In given question $a = 1$

Hence locus is $x = -1$

27. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$

(1) $(3, -2)$

(2) $(-3, 2)$

(3) $(2, -3)$

(4) $(-2, 3)$

Solution:

27) $\bar{a}\cdot\bar{b} = \bar{b}\cdot\bar{c} = \bar{c}\cdot\bar{a} = 0$

(mutually $\perp r$ vectors)

$\bar{a}\cdot\bar{c} = \lambda - 1 + 2\mu = 0 \Rightarrow \lambda + 2\mu = 1$

$\bar{b}\cdot\bar{c} = 2\lambda + 4 + \mu = 0 \Rightarrow 2\lambda + \mu = -4$

solve to get $\lambda = -3, \mu = 2$

28. The number of complex numbers z such that $|z-1| = |z+1| = |z-i|$ equals

(1) ∞

(2) 0

(3) 1

(4) 2

Solution:

28) $|z-1| = |z+1| = |z-i|$

$|z-1| = |z+1| = |z-i|$

Let $z = x+iy$

$(x-1)^2+y^2 = (x+1)^2+y^2 = x^2+(y-1)^2$

$\Rightarrow x^2+y^2-2x+1 = x^2+y^2+2x+1 = x^2+y^2-2y+1$

$\Rightarrow -2x = 2x = -2y$

① & ② $\Rightarrow x=0 \Rightarrow y=0$

$z=0$ is the only solution

No. of Solutions = 1

29. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$

(1) 2

(2) −2

(3) −1

(4) 1

Solution:

29) $x^2-x+1=0$

Multiply $(x+1)$ on both sides

$(x+1)(x^2-x+1)=0 \Rightarrow x^3+1=0$

$x^3=-1 \Rightarrow x=-1, -\omega, -\omega^2$

But $x \neq -1$ (Not given initially)

or you can solve quadratic.

$\alpha^{2009} + \beta^{2009}$

$(-\omega)^{2009} + (-\omega^2)^{2009}$

$= -\left[\omega^{2007}\cdot\omega^2 + \omega^{2\times2007}(\omega^2)^2\right]$

$= -\left[\omega^2+\omega^4\right] = -(-1) = 1$

30. Consider the following relations :

$R = \{(x, y) \mid x, y$ are real numbers and $x = wy$ for some rational number $w\}$;

$S = \{(\frac{m}{n}, \frac{p}{q}) \mid m, n, p$ and q are integers such that $n, q \neq 0$ and $qm = pn\}$.

Then

(1) R and S both are equivalence relations

(2) R is an equivalence relation but S is not an equivalence relation

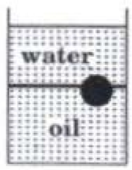
(3) neither R nor S is an equivalence relation

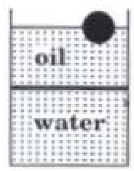
(4) S is an equivalence relation but R is not an equivalence relation

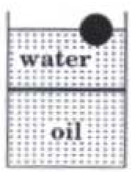
Solution:

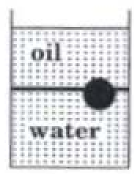
Physics

31. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position ?

(1)


(2)


(3)


(4)


Solution:

31: (4) As $\rho_{water} > \rho_{oil}$, water should be below oil.
As $\rho > \rho_{oil}$, the ball can't be in equilibrium if it is partially submerged in oil.

Directions : *Questions number 32 – 33 are based on the following paragraph.*

A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c.

32. The speed of daughter nuclei is

(1) $c\sqrt{\frac{\Delta m}{M}}$

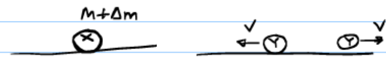
(2) $c\sqrt{\frac{\Delta m}{M + \Delta m}}$

(3) $c\frac{\Delta m}{M + \Delta m}$

(4) $c\sqrt{\frac{2\Delta m}{M}}$

Solution:

32: (4) Mass defect energy $= (M+\Delta m)c^2 - \left(\frac{M}{2}c^2\right)\cdot 2 = \Delta mc^2$ (Released in the form of K.E.)



$\Rightarrow \Delta mc^2 = \left(\frac{1}{2}\frac{M}{2}v^2\right)\cdot 2$

$\Rightarrow v = c\sqrt{\frac{2\Delta m}{M}}$

33. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then

(1) $E_2 > E_1$

(2) $E_1 = 2E_2$

(3) $E_2 = 2E_1$

(4) $E_1 > E_2$

Solution:

33: (1) Since the decay of X to Y is spontaneous, 'Y' should have a higher value of E_{bn}.

34. In a series LCR circuit $R = 200\ \Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is

(1) Zero W

(2) 242 W

(3) 305 W

(4) 210 W

Solution:

34: (2) The circuit is in resonance condition.

$P = \frac{V^2}{R} = \frac{220^2}{200} = 242\,W$

35. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto $r = R$, and $\rho(r) = 0$ for $r > R$, where r is the distance from the origin. The electric field at a distance $r\ (r < R)$ from the origin is given by

(1) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

(2) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

(3) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

(4) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

Solution:

(4) For $r < R$: $q = \int_0^r \rho\, dV = \int_0^r \rho\left(\frac{5}{4} - \frac{r}{R}\right)(4\pi r^2 dr)$

$= 4\pi\rho_0\left[\frac{5}{12}r^3 - \frac{1}{4R}r^4\right]$

Using Gauss Law: $E(4\pi r^2) = \frac{q}{\epsilon_0}$

$\Rightarrow E = \frac{\rho r}{4\epsilon_0}\left[\frac{5}{3} - \frac{r}{R}\right]$

***Directions :** Questions number 36 – 38 are based on the following paragraph.*

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

36. The speed of light in the medium is

(1) directly proportional to the intensity I

(2) maximum on the axis of the beam

(3) minimum on the axis of the beam

(4) the same everywhere in the beam

Solution:

(3) $V = \frac{c}{\mu} = \frac{c}{\mu_0 + \mu_2 I}$

I is maximum on axis. Hence, V is minimum on the axis.

37. As the beam enters the medium, it will

(1) diverge near the axis and converge near the periphery

(2) travel as a cylindrical beam

(3) diverge

(4) converge

Solution: (2)

38. The initial shape of the wavefront of the beam is

(1) convex near the axis and concave near the periphery

(2) planar

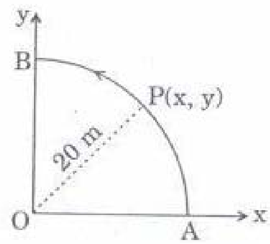
(3) convex

(4) concave

Solution:

(2) Since beam is parallel, wave front will be planar.

39. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when $t = 2$ s is nearly



(1) $7\cdot2\ m/s^2$

(2) $14\ m/s^2$

(3) $13\ m/s^2$

(4) $12\ m/s^2$

Solution:

39: (2)

$s = t^3 + 5$

$v = \frac{ds}{dt} = 3t^2$; $a_t = \frac{dv}{dt} = 6t$

$a_t = 6 \times 2 = 12\ ms^{-2}$; $a_R = \frac{v^2}{R} = \frac{12^2}{20}\ ms^{-2}$ $\Rightarrow a = \sqrt{a_R^2 + a_t^2} = 14\ ms^{-2}$

40. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density $0\cdot8\ g\ cm^{-3}$, the angle remains the same. If density of the material of the sphere is $1\cdot6\ g\ cm^{-3}$, the dielectric constant of the liquid is

(1) 2

(2) 1

(3) 4

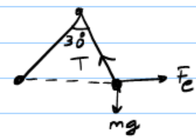
(4) 3

Solution:

(1)

In air:

$T\cos 15^\circ = mg$ & $T\sin 15^\circ = F_e$

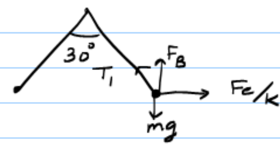


$\Rightarrow \tan 15^\circ = \dfrac{F_e}{mg}$ ... (i)

In Liquid:

$T_1 \cos 15^\circ = mg - F_B$

$T_1 \sin 15^\circ = F_e/K$



$\Rightarrow \tan 15^\circ = \dfrac{F_e/K}{mg - F_B}$

$\Rightarrow K = \dfrac{mg}{mg - F_B} = \dfrac{\rho}{\rho - \rho_\ell} = \dfrac{1.6}{1.6 - 0.8} = 2$

41. Two conductors have the same resistance at 0 °C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly

(1) $\alpha_1 + \alpha_2, \dfrac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}$

(2) $\dfrac{\alpha_1 + \alpha_2}{2}, \dfrac{\alpha_1 + \alpha_2}{2}$

(3) $\dfrac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$

(4) $\alpha_1 + \alpha_2, \dfrac{\alpha_1 + \alpha_2}{2}$

Solution:

(2) For series: $R_{eq.} = R + R$

$\Rightarrow R_{eq.}(1 + \alpha\Delta T) = R(1 + \alpha_1 \Delta T) + R(1 + \alpha_2 \Delta T)$

$\Rightarrow \alpha = \dfrac{\alpha_1 + \alpha_2}{2}$

For Parallel: $R_{eq.} = \dfrac{R \cdot R}{R + R} = \dfrac{R}{2}$

$\Rightarrow R_{eq.}(1 + \alpha\Delta T) = \dfrac{R(1 + \alpha_1 \Delta T) \cdot R(1 + \alpha_2 \Delta T)}{R(1 + \alpha_1 \Delta T) + R(1 + \alpha_2 \Delta T)}$

$\Rightarrow \alpha = \dfrac{\alpha_1 + \alpha_2}{2}$ [Taking approximation; $\alpha_1 \alpha_2 \simeq 0$, $\alpha_1 \cdot \alpha \simeq 0$, $\alpha_2 \cdot \alpha \simeq 0$]

42. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is

(1) $\frac{b^2}{4a}$

(2) $\frac{b^2}{6a}$

(3) $\frac{b^2}{2a}$

(4) $\frac{b^2}{12a}$

Solution:

(1) $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$

$U(\infty) = 0$

For equilibrium, $\frac{dU}{dx} = 0 \Rightarrow x^6 = \frac{2a}{b}$

$\Rightarrow U(eq.) = \frac{a}{(2a/b)^2} - \frac{b}{2a/b} = -\frac{b^2}{4a}$; $U_D = U(\infty) - U(eq.) = b^2/4a$

Directions : Questions number 43 – 44 contain Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

43. **Statement-1** : When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.

Statement-2 : Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.

(1) Statement-1 is false, Statement-2 is true.

(2) Statement-1 is true, Statement-2 is false.

(3) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.

(4) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** the correct explanation of Statement-1.

Solution:

43: (2) Even if the frequency of the incident radiation is fixed, photoelectrons will have different $K.E_{max}$.

44. **Statement-1 :** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.

(1) Statement-1 is false, Statement-2 is true.

(2) Statement-1 is true, Statement-2 is false.

(3) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.

(4) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** the correct explanation of Statement-1.

Solution:

(3) Since they are moving in the same direction, they must have same momentum in that direction to conserve momentum.

45. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

(1) $\frac{A-Z-12}{Z-4}$

(2) $\frac{A-Z-4}{Z-2}$

(3) $\frac{A-Z-8}{Z-4}$

(4) $\frac{A-Z-4}{Z-8}$

Solution:

(4) $Z' = Z - 2\times3 - 2\times1 = Z-8$

$A' = A - 3\times4 = A-12$

$\Rightarrow \frac{n}{p} = \frac{A'-Z'}{Z'} = \frac{(A-12)-(Z-8)}{Z-8} = \frac{A-Z-4}{Z-8}$

46. If a source of power 4 kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called

(1) microwaves

(2) γ-rays

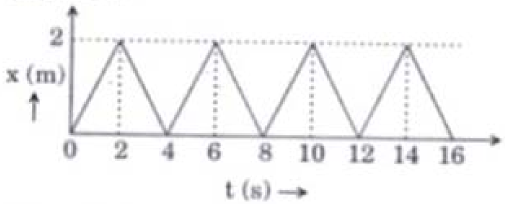
(3) X-rays

(4) ultraviolet rays

Solution:

46 : (3) $10^{20} = \frac{4\times10^3}{hc/\lambda} \Rightarrow \lambda \simeq 50\,\mathring{A}$

47. The figure shows the position – time (x – t) graph of one-dimensional motion of a body of mass 0·4 kg. The magnitude of each impulse is

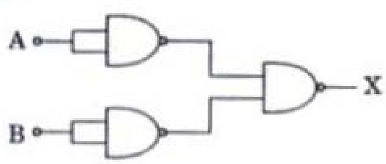


(1) 1·6 Ns
(2) 0·2 Ns
(3) 0·4 Ns
(4) 0·8 Ns

Solution:

(A) $I = \Delta p = m\Delta V = m[2 \times \text{slope of } x\text{-}t \text{ graph}] \Rightarrow I = 0.4 \times 2 \times 1 = 0.8$ Ns

48. The combination of gates shown below yields

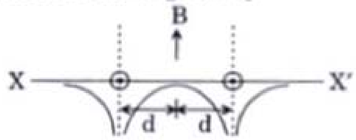


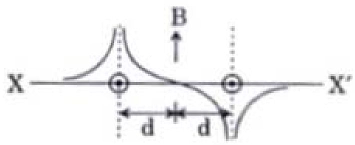
(1) XOR gate
(2) NAND gate
(3) OR gate
(4) NOT gate

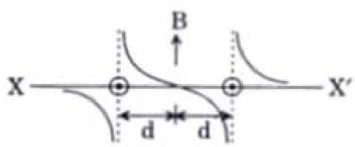
Solution:

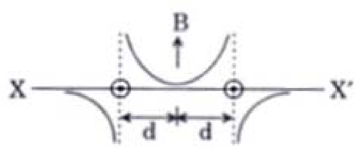
(3) $\overline{(\overline{A \cdot A}) \cdot (\overline{B \cdot B})} = \overline{\bar{A} \cdot \bar{B}} = A + B$

49. Two long parallel wires are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX' is given by

(1) 

(2) 

(3) 

(4) 

Solution: (3)

50. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1 / t_2 will be

(1) $\frac{1}{4}$

(2) 2

(3) 1

(4) $\frac{1}{2}$

Solution:

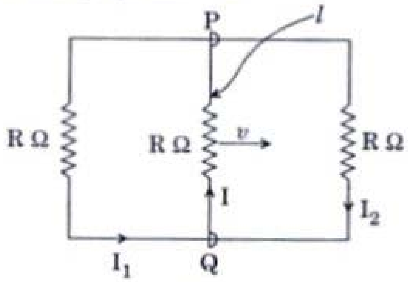
50 : (1) $E' = E/2$; $\frac{(Q')^2}{2C} = \frac{Q^2}{4C} \Rightarrow Q' = \frac{Q}{\sqrt{2}}$

$\Rightarrow Q e^{-t_1/RC} = \frac{Q}{\sqrt{2}} \Rightarrow t_1 = RC \ln\sqrt{2}$

Also, $\frac{Q}{4} = Q e^{-t_2/RC} \Rightarrow t_2 = RC \ln 4$

$\Rightarrow \frac{t_1}{t_2} = \frac{RC\ln\sqrt{2}}{RC\ln 4} = \frac{1}{4}$

51. A rectangular loop has a sliding connector PQ of length l and resistance R Ω and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



(1) $I_1 = I_2 = I = \frac{Blv}{R}$

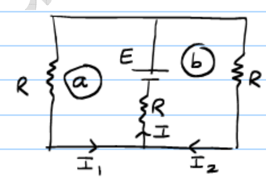
(2) $I_1 = I_2 = \frac{Blv}{6R}$, $I = \frac{Blv}{3R}$

(3) $I_1 = -I_2 = \frac{Blv}{R}$, $I = \frac{2Blv}{R}$

(4) $I_1 = I_2 = \frac{Blv}{3R}$, $I = \frac{2Blv}{3R}$

Solution:

(4) $E = B\ell v$



In loop 'a': $E = I_1R + IR$ .... (i)

In loop 'b': $E = I_2R + IR$ .... (ii)

Also, $I = I_1 + I_2$ .... (iii)

$\Rightarrow I_1 = I_2 = \frac{B\ell v}{3R} \Rightarrow I = \frac{2B\ell v}{3R}$

52. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point P (R, θ) on the circle of radius R is (Here θ is measured from the x-axis)

(1) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$

(2) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

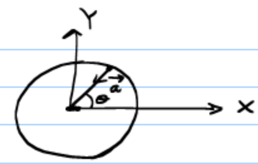
(3) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$

(4) $-\frac{v^2}{R}\sin\theta\,\hat{i} + \frac{v^2}{R}\cos\theta\,\hat{j}$

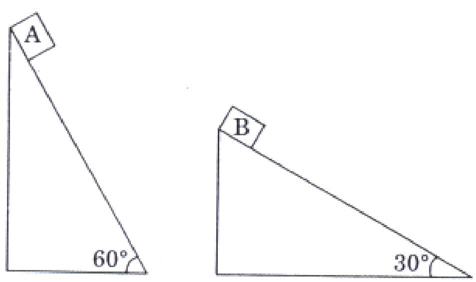
Solution:

(1) $\vec{a} = -a\cos\theta\,\hat{i} - a\sin\theta\,\hat{j}$

$= -\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$



53. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B ?



(1) Zero

(2) $4{\cdot}9\ ms^{-2}$ in vertical direction

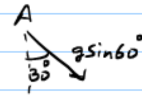
(3) $4{\cdot}9\ ms^{-2}$ in horizontal direction

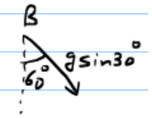
(4) $9{\cdot}8\ ms^{-2}$ in vertical direction

Solution:

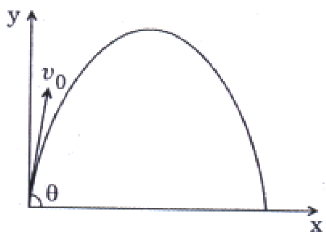
53: (2) Relative vertical acceleration :

$$a_{A/B} = g\sin 60^\circ \cos 30^\circ - g\sin 30^\circ \cos 60^\circ = \frac{3}{4}g - \frac{g}{4} = 4.9\ ms^{-2}$$





54. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is



(1) $-\frac{1}{2}\, mg\, v_0\, t^2 \cos\theta\ \hat{k}$

(2) $\frac{1}{2}\, mg\, v_0\, t^2 \cos\theta\ \hat{i}$

(3) $-\, mg\, v_0\, t^2 \cos\theta\ \hat{j}$

(4) $mg\, v_0\, t \cos\theta\ \hat{k}$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively.

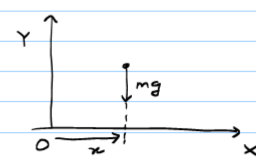
Solution:

(1) $\int \vec{\tau}_0\, dt = \vec{\Delta L}$

$\int mgx \cdot dt\, (-\hat{k}) = \vec{L}$

$\Rightarrow \vec{L} = -\int_0^t mg\, (v\cos\theta\, t)\, dt\ \hat{k}$

$\Rightarrow \vec{L} = -\frac{1}{2} mg V_0 \cos\theta\, t^2\ \hat{k}$



55. The equation of a wave on a string of linear mass density 0·04 kg m^{-1} is given by

$$y = 0{\cdot}02(\mathrm{m})\sin\left[2\pi\left(\frac{t}{0{\cdot}04(\mathrm{s})} - \frac{x}{0{\cdot}50(\mathrm{m})}\right)\right].$$

The tension in the string is

(1) 0·5 N

(2) 6·25 N

(3) 4·0 N

(4) 12·5 N

Solution:

(2) $V = \sqrt{\frac{T}{\mu}}$

$$\Rightarrow \frac{\omega^2}{k^2} = \frac{T}{\mu} \Rightarrow T = \frac{\mu\omega^2}{k^2} = \frac{0.04\left(\frac{2\pi}{0.04}\right)^2}{\left(\frac{2\pi}{0.5}\right)^2} = 6.25$$

56. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to 32 V, the efficiency of the engine is

(1) 0·99

(2) 0·25

(3) 0·5

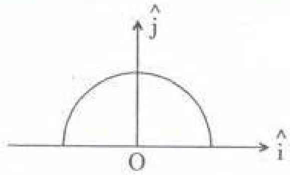
(4) 0·75

Solution:

(4) $\eta = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{4} = 0.75$

$$\left[\begin{array}{l} T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \\ \Rightarrow \frac{T_1}{T_2} = \left(\frac{1}{32}\right)^{\gamma-1} = \left(\frac{1}{32}\right)^{\frac{7}{5}-1} = \frac{1}{4} \end{array}\right]$$

57. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is



(1) $-\frac{q}{2\pi^2 \varepsilon_0 r^2}\hat{j}$

(2) $\frac{q}{2\pi^2 \varepsilon_0 r^2}\hat{j}$

(3) $\frac{q}{4\pi^2 \varepsilon_0 r^2}\hat{j}$

(4) $-\frac{q}{4\pi^2 \varepsilon_0 r^2}\hat{j}$

Solution:

(1) $\vec{E} = -\frac{\lambda}{2\pi\epsilon_0 r}\hat{j} = \frac{-Q/\pi r}{2\pi\epsilon_0 r}\hat{j} = \frac{-Q}{2\pi^2\epsilon_0 r^2}\hat{j}$

58. The respective number of significant figures for the numbers 23·023, 0·0003 and $2{\cdot}1 \times 10^{-3}$ are

(1) 5, 5, 2

(2) 4, 4, 2

(3) 5, 1, 2

(4) 5, 1, 5

Solution: 58: (3)

59. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is

(1) xy = constant

(2) $y^2 = x^2$ + constant

(3) $y = x^2$ + constant

(4) $y^2 = x$ + constant

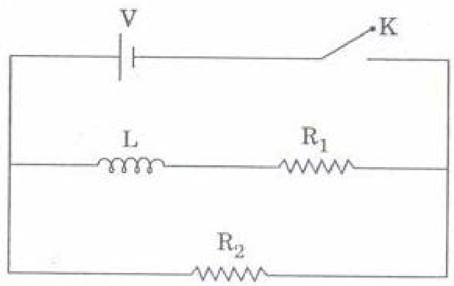
Solution:

(2) $V = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = Ky\hat{i} + Kx\hat{j}$

$\Rightarrow dx = Ky\,dt \; ; \; dy = Kx\,dt \quad \Rightarrow \frac{dy}{dx} = \frac{x}{y} \Rightarrow x\,dx = y\,dy$

$\Rightarrow y^2 = x^2 + \text{constant}$

60. In the circuit shown below, the key K is closed at t = 0. The current through the battery is



(1) $\frac{V}{R_2}$ at $t = 0$ and $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$

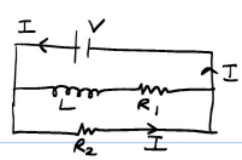
(2) $\frac{V(R_1 + R_2)}{R_1R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(3) $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(4) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1R_2}$ at $t = \infty$

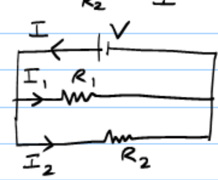
Solution:

(4) $t = 0$: $\quad I = \frac{V}{R_2}$



$t = \infty$: $\quad I = I_1 + I_2 = \frac{V_1}{R_1} + \frac{V_2}{R_2}$

$\Rightarrow I = \frac{V(R_1 + R_2)}{R_1 \cdot R_2}$



Chemistry

61. A solution containing 2·675 g of $CoCl_3 . 6\,NH_3$ (molar mass = 267·5 g mol^{-1}) is passed through a cation exchanger. The chloride ions obtained in solution were treated with excess of $AgNO_3$ to give 4·78 g of AgCl (molar mass = 143·5 g mol^{-1}). The formula of the complex is
(At. mass of Ag = 108 u)

(1) $[CoCl_3(NH_3)_3]$

(2) $[CoCl(NH_3)_5]\,Cl_2$

(3) $[Co(NH_3)_6]\,Cl_3$

(4) $[CoCl_2(NH_3)_4]\,Cl$

Solution:

61: (3) $CoCl_3 \cdot 6NH_3 + AgNO_3 \longrightarrow AgCl$

(2·675 g) ≡ 10 mmoles, $M_0 = 267.5$ gm/mol; $AgNO_3$ (excess); $AgCl$ 4·78 gm ≡ 33 moles ($M_0 = 143.5$ gm/mol)

⇒ 1 mole $CoCl_3 \cdot 6NH_3$ will react with 3 moles $AgNO_3$.

⇒ 3 Cl^- ions outside the coordination sphere.

⇒ $[Co(NH_3)_6]Cl_3$

62. The standard enthalpy of formation of NH_3 is $-46{\cdot}0$ kJ mol^{-1}. If the enthalpy of formation of H_2 from its atoms is -436 kJ mol^{-1} and that of N_2 is -712 kJ mol^{-1}, the average bond enthalpy of N – H bond in NH_3 is

(1) + 1056 kJ mol^{-1}

(2) – 1102 kJ mol^{-1}

(3) – 964 kJ mol^{-1}

(4) + 352 kJ mol^{-1}

Solution:

62: (4) $(\Delta_f H)_{NH_3} = -46\ kJ\,mol^{-1}$; $(\Delta_f H)_{H_2} = -436\ kJ\,mol^{-1}$ & $(\Delta_f H)_{N_2} = -712\ kJ\,mol^{-1}$

$$\frac{1}{2}N_2(g) + \frac{3}{2}H_2(g) \longrightarrow NH_3(g)$$

$$\Rightarrow -46 = \frac{1}{2}(712) + \frac{3}{2}(436) - 3x \quad \Rightarrow x = 352\ kJ\,mol^{-1}$$

63. The time for half life period of a certain reaction A $\longrightarrow$ Products is 1 hour. When the initial concentration of the reactant 'A', is 2·0 mol L^{-1}, how much time does it take for its concentration to come from 0·50 to 0·25 mol L^{-1} if it is a zero order reaction ?

(1) 0·25 h

(2) 1 h

(3) 4 h

(4) 0·5 h

Solution:

63: (1) A $\longrightarrow$ Products : $t_{1/2} = 1$ hour, Zero order reaction.
(Using 2 M initially)

$$2M \xrightarrow{1\,hr} 1M \xrightarrow{\frac{1}{2}\,hr} \frac{1}{2}M \xrightarrow{\frac{1}{4}\,hr} \frac{1}{4}M$$

64. If sodium sulphate is considered to be completely dissociated into cations and anions in aqueous solution, the change in freezing point of water (ΔT_f), when 0·01 mol of sodium sulphate is dissolved in 1 kg of water, is (K_f = 1·86 K kg mol^{-1})

(1) 0·0744 K

(2) 0·0186 K

(3) 0·0372 K

(4) 0·0558 K

Solution:

64: (4) Na_2SO_4 $(\alpha = 100\%)$ $\Rightarrow$ $i = 3$

$$\Delta T_f = i K_f m = 3 \times 1.86 \times 0.01 = 0.0558 \text{ K}$$

65. From amongst the following alcohols the one that would react fastest with conc. HCl and anhydrous $ZnCl_2$, is

(1) 2-Methylpropanol

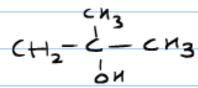
(2) 1-Butanol

(3) 2-Butanol

(4) 2-Methylpropan-2-ol

Solution:

65: (4) $ZnCl_2 + \underset{(conc.)}{HCl}$ } "Lucas test" → 3° alcohol reacts fastest.

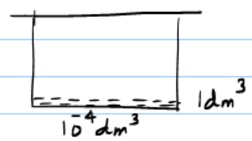


66. If 10^{-4} dm^3 of water is introduced into a 1·0 dm^3 flask at 300 K, how many moles of water are in the vapour phase when equilibrium is established ?
(Given : Vapour pressure of H_2O at 300 K is 3170 Pa; $R = 8{\cdot}314$ J K^{-1} mol^{-1})

(1) $4{\cdot}46 \times 10^{-2}$ mol
(2) $1{\cdot}27 \times 10^{-3}$ mol
(3) $5{\cdot}56 \times 10^{-3}$ mol
(4) $1{\cdot}53 \times 10^{-2}$ mol

Solution:

66: (2)

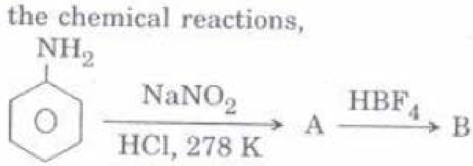


$PV = nRT$

$\Rightarrow 3170 \times 10^{-3} = n \times 8{\cdot}31 \times 300$

$\Rightarrow n = 1{\cdot}27 \times 10^{-3}$

67. In the chemical reactions,

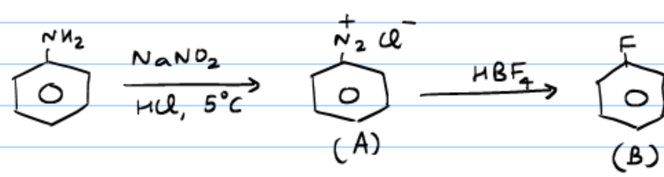


the compounds 'A' and 'B' respectively are

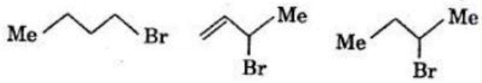
(1) benzene diazonium chloride and fluorobenzene
(2) nitrobenzene and chlorobenzene
(3) nitrobenzene and fluorobenzene
(4) phenol and benzene

Solution:

67: (1)



68. Consider the following bromides :



Ⓐ Ⓑ Ⓒ

The correct order of S_N1 reactivity is

(1) C > B > A

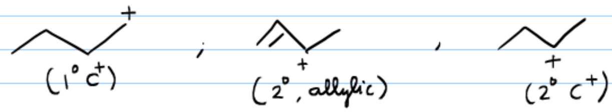
(2) A > B > C

(3) B > C > A

(4) B > A > C

Solution:

68: (3) Consider the stability of carbocation:



B > C > A

69 Which one of the following has an optical isomer ?

(1) $[Co(H_2O)_4(en)]^{3+}$

(2) $[Zn(en)_2]^{2+}$

(3) $[Zn(en)(NH_3)_2]^{2+}$

(4) $[Co(en)_3]^{3+}$

(en = ethylenediamine)

Solution:

69: (4) Optical isomerism is possible only in $[Co(en)_3]^{3+}$

70. The Gibbs energy for the decomposition of Al_2O_3 at 500 °C is as follows :

$$\frac{2}{3}Al_2O_3 \rightarrow \frac{4}{3}Al + O_2,\ \Delta_rG = +966\text{ kJ mol}^{-1}$$

The potential difference needed for electrolytic reduction of Al_2O_3 at 500 °C is at least

(1) 2·5 V

(2) 5·0 V

(3) 4·5 V

(4) 3·0 V

Solution:

70 : (1)

$$\frac{2}{3}Al_2^{3+} + 4e^- \longrightarrow \frac{4}{3}Al$$

$$\frac{2}{3}O_3^{2-} \longrightarrow O_2 + 4e^-$$

$4e^-$ transferred :

$$\Delta G^\circ = nFE^\circ \Rightarrow 966\times10^3 = 4\times96500\times E_{cell}$$

$\Rightarrow$ Pot. difference = 2.5 V

71. On mixing, heptane and octane form an ideal solution. At 373 K, the vapour pressures of the two liquid components (heptane and octane) are 105 kPa and 45 kPa respectively. Vapour pressure of the solution obtained by mixing 25·0 g of heptane and 35 g of octane will be (molar mass of heptane = 100 g mol^{-1} and of octane = 114 g mol^{-1})

(1) 96·2 kPa

(2) 144·5 kPa

(3) 72·0 kPa

(4) 36·1 kPa

Solution:

71 : (3)

Hexane (105 kPa) + Octane (45 kPa)

$25\text{ gm} = \frac{25}{100}$ moles $= 0.25$ moles

$35\text{ gm} = \frac{35}{114}$ moles $= 0.3$ moles

$$P_{Total} = 105\times\frac{0.25}{0.55} + 45\times\frac{0.3}{0.55}$$

$$= 47.27 + 24.54 = 71.8\text{ kPa}$$

72. The edge length of a face centered cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is

(1) 618 pm

(2) 144 pm

(3) 288 pm

(4) 398 pm

Solution:

72: (2) For an ionic substance with FCC : $a = 2(r^+ + r^-) \Rightarrow r^- = \frac{508}{2} - 110 = 144$ pm

73. The correct order of increasing basicity of the given conjugate bases ($R = CH_3$) is

(1) $RCO\bar{O} < \bar{N}H_2 < HC \equiv \bar{C} < \bar{R}$

(2) $RCO\bar{O} < HC \equiv \bar{C} < \bar{N}H_2 < \bar{R}$

(3) $RCO\bar{O} < HC \equiv \bar{C} < \bar{R} < \bar{N}H_2$

(4) $\bar{R} < HC \equiv \bar{C} < RCO\bar{O} < \bar{N}H_2$

Solution:

73: (2) Look for the strongest & weakest acid.

Acidic strength: $RCOOH > CH \equiv CH > NH_3 > RH$

$\Rightarrow$ Basic strength: $RCOO^- < CH \equiv C^- < \bar{N}H_2 < R^-$

74. For a particular reversible reaction at temperature T, ΔH and ΔS were found to be both +ve. If T_e is the temperature at equilibrium, the reaction would be spontaneous when

(1) T_e is 5 times T

(2) $T = T_e$

(3) $T_e > T$

(4) $T > T_e$

Solution:

74: (4) ΔH & ΔS are both +ive. ⇒ Reaction will be spontaneous at high T

⇒ Reaction will be spontaneous for $T > T_e$.

75. Consider the reaction :

$Cl_2\,(aq) + H_2S\,(aq) \rightarrow S\,(s) + 2\,H^+\,(aq) + 2\,Cl^-\,(aq)$

The rate equation for this reaction is

rate = k $[Cl_2]\,[H_2S]$

Which of these mechanisms is/are consistent with this rate equation ?

A. $Cl_2 + H_2S \rightarrow H^+ + Cl^- + Cl^+ + HS^-$ (slow)
$Cl^+ + HS^- \rightarrow H^+ + Cl^- + S$ (fast)

B. $H_2S \Leftrightarrow H^+ + HS^-$ (fast equilibrium)
$Cl_2 + HS^- \rightarrow 2\,Cl^- + H^+ + S$ (slow)

(1) Neither A nor B

(2) A only

(3) B only

(4) Both A and B

Solution:

75: (2) For (A), Rate of RDS Satisfies the given rate law.

For (B), Rate of RDS = $K_{RDS}\,[Cl_2][HS^-] = K_{RDS}\,[Cl_2]\left(\dfrac{K_{eq}\cdot[H_2S]}{[H^+]}\right)$

76. Percentages of free space in cubic close packed structure and in body centered packed structure are respectively

(1) 32% and 48%

(2) 48% and 26%

(3) 30% and 26%

(4) 26% and 32%

Solution:

76: (4) For CCP : 74% Packing fraction ⇒ 26% void
For BCC : 68% Packing fraction ⇒ 32% void

77. Out of the following, the alkene that exhibits optical isomerism is

(1) 3-methyl-1-pentene

(2) 2-methyl-2-pentene

(3) 3-methyl-2-pentene

(4) 4-methyl-1-pentene

Solution:

77: (1)

$$CH_2=CH-\overset{*}{C}H(CH_3)-CH_2CH_3$$

78. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of 44 u. The alkene is

(1) 2-butene

(2) ethene

(3) propene

(4) 1-butene

Solution:

78: (1) Alkene (Symmetrical) $\xrightarrow{O_3}$ RCHO

44 gm ⇒ $44 = x + (12+1+16)$

$RCH=CH-R$ ⇒ $x = 15$

⇒ $R = CH_3-$

79. 29·5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method and the evolved ammonia was absorbed in 20 mL of 0·1 M HCl solution. The excess of the acid required 15 mL of 0·1 M NaOH solution for complete neutralization. The percentage of nitrogen in the compound is

(1) 23·7

(2) 29·5

(3) 59·0

(4) 47·4

Solution:

79 : (1) Meq. $NH_3 = (0.1 \times 1) \times 20 - (0.1 \times 1) \times 15 = 0.5 \equiv 0.5$ mmoles of N.

$$\% = \frac{0.5 \times 14}{29.5} \times 100\% = 23.7\%$$

80. Ionisation energy of He^+ is 19.6×10^{-18} J $atom^{-1}$. The energy of the first stationary state (n = 1) of Li^{2+} is

(1) -2.2×10^{-15} J $atom^{-1}$

(2) 8.82×10^{-17} J $atom^{-1}$

(3) 4.41×10^{-16} J $atom^{-1}$

(4) -4.41×10^{-17} J $atom^{-1}$

Solution:

80 : (4) $I.E._{He^+} = +19.6 \times 10^{-18}$ J $atom^{-1}$

$$(E_1)_{Li^{2+}} = -19.6 \times 10^{-18} \times \frac{1}{2^2} \times 3^2 \text{ J atom}^{-1} = -4.41 \times 10^{-17} \text{ J atom}^{-1}$$

81. The energy required to break one mole of Cl – Cl bonds in Cl_2 is 242 kJ mol^{-1}. The longest wavelength of light capable of breaking a single Cl – Cl bond is
($c = 3 \times 10^8\ ms^{-1}$ and $N_A = 6.02 \times 10^{23}\ mol^{-1}$)

(1) 700 nm

(2) 494 nm

(3) 594 nm

(4) 640 nm

Solution:

81 : (2) $E_{\text{Per molecule}} = \frac{242 \times 10^3}{6 \times 10^{23} \times 1.6 \times 10^{-19}}\ eV = \frac{242}{96} \simeq 2.5$

$\lambda = \frac{1240}{2.5}\ nm = 494\ nm$

82. Solubility product of silver bromide is 5.0×10^{-13}. The quantity of potassium bromide (molar mass taken as 120 g mol^{-1}) to be added to 1 litre of 0.05 M solution of silver nitrate to start the precipitation of AgBr is

(1) 6.2×10^{-5} g

(2) 5.0×10^{-8} g

(3) 1.2×10^{-10} g

(4) 1.2×10^{-9} g

Solution:

82: (4) $(K_{sp})_{AgBr} = 5 \times 10^{-13}$

$\left.\begin{array}{l} KBr + \underset{1L,\ 0.05M}{AgNO_3} \end{array}\right\} \Rightarrow [Br^-] = \frac{K_{sp}}{[NO_3^-]} = \frac{5 \times 10^{-13}}{0.05} = 10^{-11}\ M = [KBr]$

$\Rightarrow g_{KBr} = 10^{-11} \times 1 \times 120 = 1.2 \times 10^{-9}\ gm$

83. The correct sequence which shows decreasing order of the ionic radii of the elements is

(1) $Na^+ > F^- > Mg^{2+} > O^{2-} > Al^{3+}$

(2) $O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$

(3) $Al^{3+} > Mg^{2+} > Na^+ > F^- > O^{2-}$

(4) $Na^+ > Mg^{2+} > Al^{3+} > O^{2-} > F^-$

Solution:

83: (2) For isoelectronic species, size decreases as 'Z' increases.

84. In aqueous solution the ionization constants for carbonic acid are

$K_1 = 4{\cdot}2 \times 10^{-7}$ and $K_2 = 4{\cdot}8 \times 10^{-11}$.

Select the correct statement for a saturated 0·034 M solution of the carbonic acid.

(1) The concentrations of H^+ and HCO_3^- are approximately equal.

(2) The concentration of H^+ is double that of CO_3^{2-}.

(3) The concentration of CO_3^{2-} is 0·034 M.

(4) The concentration of CO_3^{2-} is greater than that of HCO_3^-.

Solution:

84: (1)

$$H_2CO_3 \rightleftharpoons H^+ + HCO_3^- \qquad K_1 = 4\cdot2\times10^{-7}$$

0·034M

$0\cdot034 - x \qquad x \qquad x$

$$HCO_3^- \rightleftharpoons H^+ + CO_3^{2-} \qquad K_2 = 4\cdot8\times10^{-11}$$

$x \qquad x \qquad -$

$x-y \qquad x+y \qquad y$

$[CO_3^{2-}] \simeq K_2 = 4\cdot8\times10^{-11}$, $[H^+] = x+y \simeq x$

$[HCO_3^-] = x-y \simeq x$

85. At 25 °C, the solubility product of $Mg(OH)_2$ is $1{\cdot}0 \times 10^{-11}$. At which pH, will Mg^{2+} ions start precipitating in the form of $Mg(OH)_2$ from a solution of 0·001 M Mg^{2+} ions ?

(1) 11

(2) 8

(3) 9

(4) 10

Solution:

85 : (4) $(K_{sp})_{Mg(OH)_2} = 10^{-11} = [Mg^{2+}][OH^-]^2 = 10^{-3}[OH^-]^2 \Rightarrow [OH^-]^2 = 10^{-8}$

$\Rightarrow [OH^-] = 10^{-4}M \Rightarrow pOH = 4$

$\Rightarrow pH = 10$

86. The polymer containing strong intermolecular forces e.g. hydrogen bonding, is

(1) polystyrene

(2) natural rubber

(3) teflon

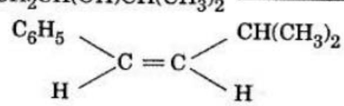
(4) nylon 6,6

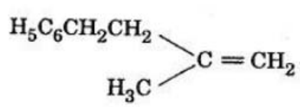
Solution:

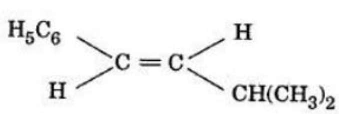
86 : (4) Nylon has very high intermolecular forces due to H- Bonding.

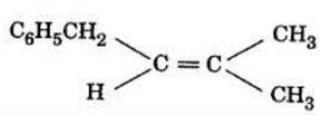
87. The main product of the following reaction is

$C_6H_5CH_2CH(OH)CH(CH_3)_2 \xrightarrow{\text{conc. } H_2SO_4} ?$

(1) 

(2) 

(3) 

(4) 

Solution:

87: (3) $C_6H_5-CH_2-\underset{OH}{CH}-\underset{CH_3}{CH}-CH_3 \xrightarrow[H_2SO_4]{Conc.} C_6H_5-CH_2-\overset{+}{C}H-\underset{CH_3}{CH}-CH_3$

$\downarrow H^-$ shift

$C_6H_5-CH=CH-\underset{CH_3}{CH}-CH_3 \xleftarrow{-H^+} C_6H_5-\overset{+}{C}H-CH_2-\underset{CH_3}{CH}-CH_3$

(Conjugated)

(trans)

88. Biuret test is ***not*** given by

(1) urea

(2) proteins

(3) carbohydrates

(4) polypeptides

Solution:

88: (3) Biuret test is given by compound having polypeptide bond which is absent in carbohydrates.

89. The correct order of $E^\circ_{M^{2+}/M}$ values with negative sign for the four successive elements Cr, Mn, Fe and Co is

(1) Fe > Mn > Cr > Co

(2) Cr > Mn > Fe > Co

(3) Mn > Cr > Fe > Co

(4) Cr > Fe > Mn > Co

Solution:

89: (3) Order of reduction potential is : Mn > Cr > Fe > Co

90. Three reactions involving $H_2PO_4^-$ are given below :

(i) $H_3PO_4 + H_2O \rightarrow H_3O^+ + H_2PO_4^-$

(ii) $H_2PO_4^- + H_2O \rightarrow HPO_4^{2-} + H_3O^+$

(iii) $H_2PO_4^- + OH^- \rightarrow H_3PO_4 + O^{2-}$

In which of the above does $H_2PO_4^-$ act as an acid ?

(1) (iii) only

(2) (i) only

(3) (ii) only

(4) (i) and (ii)

Solution:

90: (3) Acid : loss of H^+ ion :

$$H_2PO_4^- + H_2O \longrightarrow HPO_4^{2-} + H_3O^+$$