Comprehensive Test Series-01

(Differentiation)

XII

TIME: 2hr. **MM: 60**

General Instructions:

- All Questions are compulsory.
- Use of calculator is not permitted.

Differentiate the functions with respect to x.

Q.1 Cos $x^3 . \sin^2 (x^5)$ (3)

Q2. $2\sqrt{\cot(x^2)}$ (3)

Find $\frac{dy}{dx}$ in the following:

Q3. $x^3 + x^2y + xy^2 + y^3 = 81$ (3)

Q4. $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0 < x < 1$ (3)

Q5. $\sqrt{e^{\sqrt{x}}}$, $x > 0$ (3)

Q6. $x^{\sin x} + (\sin x)^{\cos x}$ (4)

If x and y are connected parametrically by the equations given.

Q7. $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$ (3)

Q8. $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$ (4)

Q9. If $x = \sqrt{a^{\sin^{-1} t}}, y = \sqrt{a^{\cos^{-1} t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$ (3)

Find the second order derivatives of the functions

Q10. If $e^y (x + 1) == 1$, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ (3)

Q11. If $y = (\tan^{-1} x)^2$, show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1 = 2$ (3)

Q12. Verify Rolle's theorem for the function $y = x^2 + 2$, $a = -2$ and $b = 2$. (3)

Q13. Verify Mean Value Theorem for the function $f(x) = x^2$ in the interval [2, 4]. (3)

Differentiate w.r.t. x the function.

Q14. $\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right], 0 < x < \frac{\pi}{2}$ (4)

Q15. Find $\frac{dy}{dx}$, if $y = \sin^{-1} x + \sin^{-1} \sqrt{1-x^2}$, $-1 \leq x \leq 1$. (4)

Q16. If $\cos y = x \cos(a + y)$ with $\cos a \neq \pm$, prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$. (4)

Q17. If $y = e^{a\cos^{-1} x}$, $-1 \leq x \leq 1$, show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2 y = 0$. Differentiate the following w.r.t. x. (4)

Q18. $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ (3)